CHAPTER 6

Systems of Equations and Inequalities

Where's *the Money?*

You can solve a system of equations to determine how many basketball game tickets you can buy at different price levels.

go.hrw.com
Chapter Project Online
KEYWORD: MA7 ChProj

ARE YOU READY?

✓ Vocabulary

Match each term on the left with a definition on the right.

1. inequality
2. linear equation
3. ordered pair
4. slope
5. solution of an equation

A. a pair of numbers (x, y) that represent the coordinates of a point

B. a statement that two quantities are not equal

C. the y-value of the point at which the graph of an equation crosses the y-axis

D. a value of the variable that makes the equation true

E. the ratio of the vertical change to the horizontal change for a nonvertical line

F. an equation whose graph is a straight line

✓ Graph Linear Functions

Graph each function.

6. $y = \frac{3}{4}x + 1$
7. $y = -3x + 5$
8. $y = x - 6$
9. $x + y = 4$
10. $y = -\frac{2}{3}x + 4$
11. $y = -5$

✓ Solve Multi-Step Equations

Solve each equation.

12. $-7x - 18 = 3$
13. $12 = -3n + 6$
14. $\frac{1}{2}d + 30 = 32$
15. $-2p + 9 = -3$
16. $33 = 5y + 8$
17. $-3 + 3x = 27$

✓ Solve for a Variable

Solve each equation for y.

18. $7x + y = 4$
19. $y + 2 = -4x$
20. $8 = x - y$
21. $x + 2 = y - 5$
22. $2y - 3 = 12x$
23. $y + \frac{3}{4}x = 4$

✓ Evaluate Expressions

Evaluate each expression for the given value of the variable.

24. $t - 5$ for $t = 7$
25. $9 - 2a$ for $a = 4$
26. $\frac{1}{2}x - 2$ for $x = 14$
27. $n + 15$ for $n = 37$
28. $9c + 4$ for $c = \frac{1}{3}$
29. $16 + 3d$ for $d = 5$

✓ Solve and Graph Inequalities

Solve and graph each inequality.

30. $b - 9 \geq 1$
31. $-2x < 10$
32. $3y \leq -3$
33. $\frac{1}{3}y \leq 5$

Study Guide: Preview

Where You've Been

Previously, you
- solved one-step and multi-step equations.
- solved one-step and multi-step inequalities.
- graphed linear equations on a coordinate plane.

In This Chapter

You will study
- how to find a solution that satisfies two linear equations.
- how to find solutions that satisfy two linear inequalities.
- how to graph one or more linear inequalities on a coordinate plane.

Where You're Going

You can use the skills in this chapter
- to determine which purchases are better deals.
- in other classes, such as Economics and Chemistry.
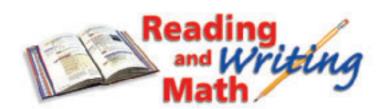- to solve linear equations that involve three or more variables in future math classes.

Key Vocabulary/Vocabulario

consistent system	sistema consistente
dependent system	sistema dependiente
inconsistent system	sistema inconsistente
independent system	sistema independiente
linear inequality	desigualdad lineal
solution of a linear inequality	solución de una desigualdad lineal
system of linear equations	sistema de ecuaciones lineales

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The word *system* means "a group." How do you think a **system of linear equations** is different from a linear equation?

2. A **consistent system** has *at least one* solution. How many solutions do you think an **inconsistent system** has?

3. A **dependent system** has infinitely many solutions. Which vocabulary term above means a system with *exactly one* solution?

4. In Chapters 4 and 5, you saw that a solution of a linear equation was the ordered pair that made the equation true. Modify this to define **solution of a linear inequality** .

Reading and Writing Math

Writing Strategy: Write a Convincing Argument/Explanation

The Write About It icon appears throughout the book. These icons identify questions that require you to write a complete argument or explanation. Writing a convincing argument or explanation shows that you have a solid understanding of a concept.

To be effective, an argument or explanation should include

- evidence, work, or facts.
- a complete response that will answer or explain.

From Lesson 2-9

23. Write About It Lewis invested $1000 at 3% simple interest for 4 years. Lisa invested $1000 at 4% simple interest for 3 years. Explain why Lewis and Lisa earned the same amount of interest.

Step 1 **Identify what you need to answer or explain.**
Explain why Lewis and Lisa earned the same amount of interest.

Step 2 **Give evidence, work, or facts that are needed to answer the question.**
Use the formula for simple interest to find the amount of interest earned: $I = Prt$.
Lewis: $P = 1000, r = 0.03, t = 4$ **Lisa:** $P = 1000, r = 0.04, t = 3$

$$I = Prt = 1000(0.03)(4) = 120 \qquad I = Prt = 1000(0.04)(3) = 120$$

$$I = 1000(0.12) = \$120 \qquad\qquad I = 1000(0.12) = \$120$$

Step 3 **Write a complete response that answers or explains.**
Lewis and Lisa both invested the same amount of money, $1000. They earned the same amount of interest because 0.04×3 and 0.03×4 both equal 0.12. They both earned $0.12 \times \$1000$, or $120.

Try This

Write a convincing argument or explanation.

1. What is the least whole number that is a solution of $12x + 15.4 > 118.92$? Explain.

2. Which equation has an error? Explain the error.

   A. $4(6 \cdot 5) = (4)6 \cdot (4)5$ **B.** $4(6 \cdot 5) = (4 \cdot 6)5$

Solve Linear Equations by Using a Spreadsheet

You can use a spreadsheet to answer "What if…?" questions. By changing one or more values, you can quickly model different scenarios.

Use with Lesson 6-1

Activity

Company Z makes DVD players. The company's costs are $400 per week plus $20 per DVD player. Each DVD player sells for $45. How many DVD players must company Z sell in one week to make a profit?

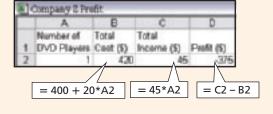Let n represent the number of DVD players company Z sells in one week.

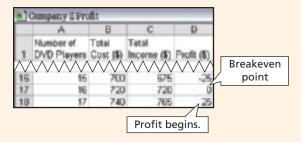|	
$c = 400 + 20n$	*The total cost is $400 plus $20 times the number of DVD players made.*
$s = 45n$	*The total sales income is $45 times the number of DVD players sold.*
$p = s - c$	*The total profit is the sales income minus the total cost.*

1 Set up your spreadsheet with columns for number of DVD players, total cost, total income, and profit.

2 Under Number of DVD Players, enter 1 in cell A2.

3 Use the equations above to enter the formulas for total cost, total sales, and total profit in row 2.

 • In cell B2, enter the formula for total cost.

 • In cell C2, enter the formula for total sales income.

 • In cell D2, enter the formula for total profit.

	A	B	C	D
1	Number of DVD Players	Total Cost ($)	Total Income ($)	Profit ($)
2	1	420	45	-375

= 400 + 20*A2   = 45*A2   = C2 − B2

4 Fill columns A, B, C, and D by selecting cells A1 through D1, clicking the small box at the bottom right corner of cell D2, and dragging the box down through several rows.

5 Find the point where the profit is $0. This is known as the breakeven point, where total cost and total income are the same.

	A	B	C	D
1	Number of DVD Players	Total Cost ($)	Total Income ($)	Profit ($)
15	15	700	675	-25
16	16	720	720	0
17	17	740	765	25

Breakeven point

Profit begins.

Company Z must sell 17 DVD players to make a profit. The profit is $25.

Try This

For Exercises 1 and 2, use the spreadsheet from the activity.

1. If company Z sells 10 DVD players, will they make a profit? Explain. What if they sell 16?

2. Company Z makes a profit of $225 dollars. How many DVD players did they sell?

For Exercise 3, make a spreadsheet.

3. Company Y's costs are $400 per week plus $20 per DVD player. They want the breakeven point to occur with sales of 8 DVD players. What should the sales price be?

Solving Systems by Graphing

Objectives
Identify solutions of systems of linear equations in two variables.

Solve systems of linear equations in two variables by graphing.

Vocabulary
system of linear equations
solution of a system of linear equations

Why learn this?
You can compare costs by graphing a system of linear equations. (See Example 3.)

Sometimes there are different charges for the same service or product at different places. For example, Bowl-o-Rama charges $2.50 per game plus $2 for shoe rental while Bowling Pinz charges $2 per game plus $4 for shoe rental. A *system of linear equations* can be used to compare these charges.

A **system of linear equations** is a set of two or more linear equations containing two or more variables. A **solution of a system of linear equations** with two variables is an ordered pair that satisfies each equation in the system. So, if an ordered pair is a solution, it will make both equations true.

EXAMPLE 1 Identifying Solutions of Systems

Tell whether the ordered pair is a solution of the given system.

A $(4, 1)$; $\begin{cases} x + 2y = 6 \\ x - y = 3 \end{cases}$

$x + 2y = 6$	
$(4) + 2(1)$	6
$4 + 2$	6
6	6 ✓

$x - y = 3$	
$(4) - (1)$	3
3	3 ✓

Substitute 4 for x and 1 for y in each equation in the system.

The ordered pair $(4, 1)$ makes both equations true.

$(4, 1)$ is a solution of the system.

Helpful Hint

If an ordered pair does not satisfy the first equation in the system, there is no need to check the other equations.

B $(-1, 2)$; $\begin{cases} 2x + 5y = 8 \\ 3x - 2y = 5 \end{cases}$

$2x + 5y = 8$	
$2(-1) + 5(2)$	8
$-2 + 10$	8
8	8 ✓

$3x - 2y = 5$	
$3(-1) - 2(2)$	5
$-3 - 4$	5
-7	5 ✗

Substitute −1 for x and 2 for y in each equation in the system.

The ordered pair $(-1, 2)$ makes one equation true, but not the other.

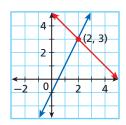$(-1, 2)$ is not a solution of the system.

CHECK IT OUT! Tell whether the ordered pair is a solution of the given system.

1a. $(1, 3)$; $\begin{cases} 2x + y = 5 \\ -2x + y = 1 \end{cases}$

1b. $(2, -1)$; $\begin{cases} x - 2y = 4 \\ 3x + y = 6 \end{cases}$

All solutions of a linear equation are on its graph. To find a solution of a system of linear equations, you need a point that each line has in common. In other words, you need their point of intersection.

$$\begin{cases} y = 2x - 1 \\ y = -x + 5 \end{cases}$$

The point $(2, 3)$ is where the two lines intersect and is a solution of both equations, so $(2, 3)$ is the solution of the system.
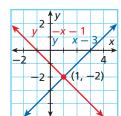

EXAMPLE 2 Solving a System of Linear Equations by Graphing

Solve each system by graphing. Check your answer.

A $\begin{cases} y = x - 3 \\ y = -x - 1 \end{cases}$

Graph the system.
The solution appears to be at $(1, -2)$.

Check
Substitute $(1, -2)$ into the system.

$y = x - 3$		$y = -x - 1$	
(-2)	$(1) - 3$	(-2)	$-(1) - 1$
-2	-2 ✓	-2	-2 ✓

$(1, -2)$ is a solution of the system.
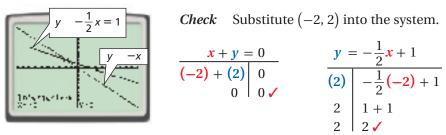
B $\begin{cases} x + y = 0 \\ y = -\dfrac{1}{2}x + 1 \end{cases}$

$$\begin{array}{r} x + y = 0 \\ \underline{-x \qquad -x} \\ y = -x \end{array}$$

Rewrite the first equation in slope-intercept form.

Graph using a calculator and then use the intersection command.

Check Substitute $(-2, 2)$ into the system.

$x + y = 0$		$y = -\dfrac{1}{2}x + 1$	
$(-2) + (2)$	0	(2)	$-\dfrac{1}{2}(-2) + 1$
0	0 ✓	2	$1 + 1$
		2	2 ✓

The solution is $(-2, 2)$.

CHECK IT OUT! Solve each system by graphing. Check your answer.

2a. $\begin{cases} y = -2x - 1 \\ y = x + 5 \end{cases}$

2b. $\begin{cases} y = \dfrac{1}{3}x - 3 \\ 2x + y = 4 \end{cases}$

> **Helpful Hint**
>
> Sometimes it is difficult to tell exactly where the lines cross when you solve by graphing. It is good to confirm your answer by substituting it into both equations.

EXAMPLE 3

Problem-Solving Application

Bowl-o-Rama charges $2.50 per game plus $2 for shoe rental, and Bowling Pinz charges $2 per game plus $4 for shoe rental. For how many games will the cost to bowl be the same at both places? What is that cost?

PROBLEM SOLVING

1. Understand the Problem

The **answer** will be the number of games played for which the total cost is the same at both bowling alleys. **List the important information:**
- Game price: Bowl-o-Rama $2.50   Bowling Pinz: $2
- Shoe-rental fee: Bowl-o-Rama $2   Bowling Pinz: $4

2. Make a Plan

Write a system of equations, one equation to represent the price at each company. Let x be the number of games played and y be the total cost.

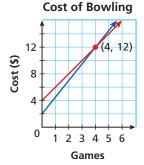|	Total cost	is	price per game	times	games	plus	shoe rental.
Bowl-o-Rama	y	=	2.5	•	x	+	2
Bowling Pinz	y	=	2	•	x	+	4

3. Solve

Graph $y = 2.5x + 2$ and $y = 2x + 4$. The lines appear to intersect at $(4, 12)$. So, the cost at both places will be the same for 4 games bowled and that cost will be $12.

Cost of Bowling

4. Look Back

Check $(4, 12)$ using both equations.
Cost of bowling 4 games at Bowl-o-Rama:
$2.5(4) + $2 = 10 + 2 = 12$ ✓
Cost of bowling 4 games at Bowling Pinz:
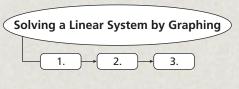$2(4) + $4 = 8 + 4 = 12$ ✓

CHECK IT OUT!

3. Video club A charges $10 for membership and $3 per movie rental. Video club B charges $15 for membership and $2 per movie rental. For how many movie rentals will the cost be the same at both video clubs? What is that cost?

THINK AND DISCUSS

1. Explain how to use a graph to solve a system of linear equations.

2. Explain how to check a solution of a system of linear equations.

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write a step for solving a linear system by graphing. More boxes may be added.

Know it! Note

Solving a Linear System by Graphing

1. → 2. → 3.

6-1

Exercises

go.hrw.com
Homework Help Online
KEYWORD: MA7 6-1
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

1. **Vocabulary** Describe a *solution of a system of linear equations*.

SEE EXAMPLE 1
p. 383

Tell whether the ordered pair is a solution of the given system.

2. $(2, -2)$; $\begin{cases} 3x + y = 4 \\ x - 3y = -4 \end{cases}$

3. $(3, -1)$; $\begin{cases} x - 2y = 5 \\ 2x - y = 7 \end{cases}$

4. $(-1, 5)$; $\begin{cases} -x + y = 6 \\ 2x + 3y = 13 \end{cases}$

SEE EXAMPLE 2
p. 384

Solve each system by graphing. Check your answer.

5. $\begin{cases} y = \frac{1}{2}x \\ y = -x + 3 \end{cases}$

6. $\begin{cases} y = x - 2 \\ 2x + y = 1 \end{cases}$

7. $\begin{cases} -2x - 1 = y \\ x + y = 3 \end{cases}$

SEE EXAMPLE 3
p. 385

8. To deliver mulch, Lawn and Garden charges $30 per cubic yard of mulch plus a $30 delivery fee. Yard Depot charges $25 per cubic yard of mulch plus a $55 delivery fee. For how many cubic yards will the cost be the same? What will that cost be?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
9–11	1
12–15	2
16	3

Extra Practice
Skills Practice p. S14
Application Practice p. S33

Tell whether the ordered pair is a solution of the given system.

9. $(1, -4)$; $\begin{cases} x - 2y = 8 \\ 4x - y = 8 \end{cases}$

10. $(-2, 1)$; $\begin{cases} 2x - 3y = -7 \\ 3x + y = -5 \end{cases}$

11. $(5, 2)$; $\begin{cases} 2x + y = 12 \\ -3y - x = -11 \end{cases}$

Solve each system by graphing. Check your answer.

12. $\begin{cases} y = \frac{1}{2}x + 2 \\ y = -x - 1 \end{cases}$

13. $\begin{cases} y = x \\ y = -x + 6 \end{cases}$

14. $\begin{cases} -2x - 1 = y \\ x = -y + 3 \end{cases}$
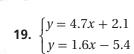
15. $\begin{cases} x + y = 2 \\ y = x - 4 \end{cases}$

16. **Multi-Step** Angelo runs 7 miles per week and increases his distance by 1 mile each week. Marc runs 4 miles per week and increases his distance by 2 miles each week. In how many weeks will Angelo and Marc be running the same distance? What will that distance be?

17. **School** The school band sells carnations on Valentine's Day for $2 each. They buy the carnations from a florist for $0.50 each, plus a $16 delivery charge.
   a. Write a system of equations to describe the situation.
   b. Graph the system. What does the solution represent?
   c. Explain whether the solution shown on the graph makes sense in this situation. If not, give a reasonable solution.

MULTI-STEP TEST PREP

18. This problem will prepare you for the Multi-Step Test Prep on page 412.
   a. The Warrior baseball team is selling hats as a fund-raiser. They contacted two companies. Hats Off charges a $50 design fee and $5 per hat. Top Stuff charges a $25 design fee and $6 per hat. Write an equation for each company's pricing.
   b. Graph the system of equations from part **a.** For how many hats will the cost be the same? What is that cost?
   c. Explain when it is cheaper for the baseball team to use Top Stuff and when it is cheaper to use Hats Off.

Graphing Calculator Use a graphing calculator to graph and solve the systems of equations in Exercises 19–22. Round your answer to the nearest tenth.

19. $\begin{cases} y = 4.7x + 2.1 \\ y = 1.6x - 5.4 \end{cases}$

20. $\begin{cases} 4.8x + 0.6y = 4 \\ y = -3.2x + 2.7 \end{cases}$

21. $\begin{cases} y = \dfrac{5}{4}x - \dfrac{2}{3} \\ \dfrac{8}{3}x + y = \dfrac{5}{9} \end{cases}$

22. $\begin{cases} y = 6.9x + 12.4 \\ y = -4.1x - 5.3 \end{cases}$

Landscaping

23. Landscaping The gardeners at Middleton Place Gardens want to plant a total of 45 white and pink hydrangeas in one flower bed. In another flower bed, they want to plant 120 hydrangeas. In this bed, they want 2 times the number of white hydrangeas and 3 times the number of pink hydrangeas as in the first bed. Use a system of equations to find how many white and how many pink hydrangeas the gardeners should buy altogether.

24. Fitness Rusty burns 5 Calories per minute swimming and 11 Calories per minute jogging. In the morning, Rusty burns 200 Calories walking and swims for x minutes. In the afternoon, Rusty will jog for x minutes. How many minutes must he jog to burn at least as many Calories y in the afternoon as he did in the morning? Round your answer up to the next whole number of minutes.

Middleton Place Gardens, South Carolina, are the United States' oldest landscaped gardens. The gardens were established in 1741 and opened to the public in the 1920s.

25. A tree that is 2 feet tall is growing at a rate of 1 foot per year. A 6-foot tall tree is growing at a rate of 0.5 foot per year. In how many years will the trees be the same height?

26. Critical Thinking Write a real-world situation that could be represented by the system $\begin{cases} y = 3x + 10 \\ y = 5x + 20 \end{cases}$.

27. Write About It When you graph a system of linear equations, why does the intersection of the two lines represent the solution of the system?

TEST PREP

28. Taxi company A charges $4 plus $0.50 per mile. Taxi company B charges $5 plus $0.25 per mile. Which system best represents this problem?

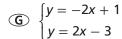Ⓐ $\begin{cases} y = 4x + 0.5 \\ y = 5x + 0.25 \end{cases}$

Ⓒ $\begin{cases} y = -4x + 0.5 \\ y = -5x + 0.25 \end{cases}$

Ⓑ $\begin{cases} y = 0.5x + 4 \\ y = 0.25x + 5 \end{cases}$

Ⓓ $\begin{cases} y = -0.5x + 4 \\ y = -0.25x + 5 \end{cases}$

29. Which system of equations represents the given graph?

Ⓕ $\begin{cases} y = 2x - 1 \\ y = \dfrac{1}{3}x + 3 \end{cases}$

Ⓗ $\begin{cases} y = 2x + 1 \\ y = \dfrac{1}{3}x - 3 \end{cases}$

Ⓖ $\begin{cases} y = -2x + 1 \\ y = 2x - 3 \end{cases}$

Ⓙ $\begin{cases} y = -2x - 1 \\ y = 3x - 3 \end{cases}$

30. Gridded Response Which value of b will make the system $y = 2x + 2$ and $y = 2.5x + b$ intersect at the point $(2, 6)$?

CHALLENGE AND EXTEND

31. **Entertainment** If the pattern in the table continues, in what month will the number of sales of VCRs and DVD players be the same? What will that number be?

Total Number Sold				
Month	1	2	3	4
VCRs	500	490	480	470
DVD Players	250	265	280	295

32. Long Distance Inc. charges a $1.45 connection charge and $0.03 per minute. Far Away Calls charges a $1.52 connection charge and $0.02 per minute.

   a. For how many minutes will a call cost the same from both companies? What is that cost?

   b. When is it better to call using Long Distance Inc.? Far Away Calls? Explain.

   c. **What if...?** Long Distance Inc. raised its connection charge to $1.50 and Far Away Calls decreased its connection charge by 2 cents. How will this affect the graphs? Now which company is better to use for calling long distance? Why?

SPIRAL REVIEW

Solve each equation. *(Lesson 2-2)*

33. $18 = \frac{3}{7}x$ 34. $-\frac{x}{5} = 12$ 35. $-6y = -13.2$ 36. $\frac{2}{5} = \frac{y}{12}$

Describe the solutions of each inequality in words. *(Lesson 3-1)*

37. $3c < 15$ 38. $\frac{1}{3}x \geq 9$ 39. $5 + a > 11$

Solve each inequality and graph the solutions. *(Lesson 3-4)*

40. $4(2x + 1) > 28$ 41. $3^3 + 9 \leq -4c$ 42. $\frac{1}{8}x + \frac{3}{5} \leq \frac{3}{8}$

Career Path

go.hrw.com
Career Resources Online
KEYWORD: MA7 Career

Q: What math classes did you take in high school?

A: Career Math, Algebra, and Geometry

Q: What are you studying and what math classes have you taken?

A: I am really interested in aviation. I am taking Statistics and Trigonometry. Next year I will take Calculus.

Q: How is math used in aviation?

A: I use math to interpret aeronautical charts. I also perform calculations involving wind movements, aircraft weight and balance, and fuel consumption. These skills are necessary for planning and executing safe air flights.

Q: What are your future plans?

A: I could work as a commercial or corporate pilot or even as a flight instructor. I could also work toward a bachelor's degree in aviation management, air traffic control, aviation electronics, aviation maintenance, or aviation computer science.

Ethan Reynolds
Applied Sciences major

Algebra LAB

Model Systems of Linear Equations

You can use algebra tiles to model and solve some systems of linear equations.

Use with Lesson 6-2

KEY

$\boxed{+} = 1$

$\boxed{-} = -1$

$\boxed{+} = x$ $\boxed{-} = -x$

REMEMBER

When two expressions are equal, you can substitute one for the other in any expression or equation.

go.hrw.com
Lab Resources Online
KEYWORD: MA7 Lab6

Activity

Use algebra tiles to model and solve $\begin{cases} y = 2x - 3 \\ x + y = 9 \end{cases}$.

MODEL		ALGEBRA
x y 9	*The first equation is solved for y. Model the second equation, x + y = 9, by substituting 2x − 3 for y.*	$x + y = 9$ $x + (2x - 3) = 9$ $3x - 3 = 9$
	Add 3 yellow tiles on both sides of the mat. This represents adding 3 to both sides of the equation. *Remove zero pairs.*	$\begin{array}{rcr} 3x - 3 = & 9 \\ +3 & +3 \\ \hline 3x & = & 12 \end{array}$
	Divide each group into 3 equal groups. Align one x-tile with each group on the right side. One x-tile is equivalent to 4 yellow tiles. x = 4	$\dfrac{3x}{3} = \dfrac{12}{3}$ $x = 4$

To solve for *y*, substitute 4 for *x* in one of the equations: $\begin{aligned} y &= 2x - 3 \\ &= 2(4) - 3 \\ &= 5 \end{aligned}$

The solution is (4, 5).

Try This

Model and solve each system of equations.

1. $\begin{cases} y = x + 3 \\ 2x + y = 6 \end{cases}$
2. $\begin{cases} 2x + 3 = y \\ x + y = 6 \end{cases}$
3. $\begin{cases} 2x + 3y = 1 \\ x = -1 - y \end{cases}$
4. $\begin{cases} y = x + 1 \\ 2x - y = -5 \end{cases}$

Solving Systems by Substitution

Objective
Solve systems of linear equations in two variables by substitution.

Why learn this?
You can solve systems of equations to help select the best value among high-speed Internet providers. (See Example 3.)

CAMPING OUT FOR THE BEST TICKETS ISN'T WHAT IT USED TO BE...

Sometimes it is difficult to identify the exact solution to a system by graphing. In this case, you can use a method called *substitution*.

The goal when using substitution is to reduce the system to one equation that has only one variable. Then you can solve this equation by the methods taught in Chapter 2.

Know it!
Note

Solving Systems of Equations by Substitution
Step 1 Solve for one variable in at least one equation, if necessary.
Step 2 Substitute the resulting expression into the other equation.
Step 3 Solve that equation to get the value of the first variable.
Step 4 Substitute that value into one of the original equations and solve.
Step 5 Write the values from Steps 3 and 4 as an ordered pair, (x, y), and check.

EXAMPLE 1 **Solving a System of Linear Equations by Substitution**

Solve each system by substitution.

A $\begin{cases} y = 2x \\ y = x + 5 \end{cases}$

Helpful Hint
You can substitute the value of one variable into *either* of the original equations to find the value of the other variable.

Step 1 $y = 2x$ *Both equations are solved for y.*
        $y = x + 5$

Step 2 $y = x + 5$ *Substitute 2x for y in the second equation.*
        $2x = x + 5$

Step 3 $\underline{-x -x}$ *Solve for x. Subtract x from both sides to combine*
        $x = 5$ *like terms.*

Step 4 $y = 2x$ *Write one of the original equations.*
        $y = 2(5)$ *Substitute 5 for x.*
        $y = 10$

Step 5 $(5, 10)$ *Write the solution as an ordered pair.*

Check Substitute $(5, 10)$ into both equations in the system.

$y = 2x$			$y = x + 5$	
10	2(5)		10	5 + 5
10	10 ✓		10	10 ✓

Solve each system by substitution.

B $\begin{cases} 2x + y = 5 \\ y = x - 4 \end{cases}$

Step 1 $y = x - 4$ *The second equation is solved for y.*

Step 2 $2x + y = 5$

$2x + (x - 4) = 5$ *Substitute x − 4 for y in the first equation.*
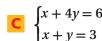
Step 3 $3x - 4 = 5$ *Simplify. Then solve for x.*

$\underline{+4 \quad +4}$ *Add 4 to both sides.*

$3x = 9$

$\dfrac{3x}{3} = \dfrac{9}{3}$ *Divide both sides by 3.*

$x = 3$

Step 4 $y = x - 4$ *Write one of the original equations.*

$y = 3 - 4$ *Substitute 3 for x.*

$y = -1$

Step 5 $(3, -1)$ *Write the solution as an ordered pair.*

C $\begin{cases} x + 4y = 6 \\ x + y = 3 \end{cases}$

Step 1 $x + 4y = 6$ *Solve the first equation for x by subtracting*

$\underline{-4y \quad -4y}$ *4y from both sides.*

$x = 6 - 4y$

Step 2 $x + y = 3$

$(6 - 4y) + y = 3$ *Substitute 6 − 4y for x in the second equation.*

Step 3 $6 - 3y = 3$ *Simplify. Then solve for y.*

$\underline{-6 \qquad -6}$ *Subtract 6 from both sides.*

$-3y = -3$

$\dfrac{-3y}{-3} = \dfrac{-3}{-3}$ *Divide both sides by −3.*

$y = 1$

Step 4 $x + y = 3$ *Write one of the original equations.*

$x + 1 = 3$ *Substitute 1 for y.*

$\underline{-1 \quad -1}$ *Subtract 1 from both sides.*

$x = 2$

Step 5 $(2, 1)$ *Write the solution as an ordered pair.*

Helpful Hint

Sometimes neither equation is solved for a variable. You can begin by solving either equation for either *x* or *y*.

CHECK IT OUT! **Solve each system by substitution.**

1a. $\begin{cases} y = x + 3 \\ y = 2x + 5 \end{cases}$ **1b.** $\begin{cases} x = 2y - 4 \\ x + 8y = 16 \end{cases}$ **1c.** $\begin{cases} 2x + y = -4 \\ x + y = -7 \end{cases}$

Sometimes you substitute an expression for a variable that has a coefficient. When solving for the second variable in this situation, you can use the Distributive Property.

EXAMPLE **2**   **Using the Distributive Property**

Solve $\begin{cases} 4y - 5x = 9 \\ x - 4y = 11 \end{cases}$ by substitution.

Step 1 $x - 4y = \quad 11$

$\underline{\quad + 4y \quad + 4y}$

$x \qquad = \quad 4y + 11$

Solve the second equation for x by adding 4y to each side.

Step 2 $\qquad\qquad 4y - 5x = 9$

$4y - 5(4y + 11) = 9$

Substitute 4y + 11 for x in the first equation.

Step 3 $4y - 5(4y) - 5(11) = 9$

$4y - 20y - 55 = 9$

$-16y - 55 = \quad 9$

$\underline{\qquad\qquad + 55 \quad + 55}$

$-16y \qquad = \quad 64$

$\dfrac{-16y}{-16} = \dfrac{64}{-16}$

$y = -4$

Distribute −5 to the expression in the parentheses. Simplify. Solve for y.

Add 55 to both sides.

Divide both sides by −16.

Step 4 $\qquad x - 4y = 11$

$x - 4(-4) = 11$

$x + 16 = \quad 11$

$\underline{\qquad - 16 \quad - 16}$

$x \qquad = \quad -5$

Write one of the original equations.

Substitute −4 for y.

Simplify.

Subtract 16 from both sides.

Step 5 $(-5, -4)$

Write the solution as an ordered pair.

Caution!

When you solve one equation for a variable, you must substitute the value or expression into the *other* original equation, not the one that has just been solved.

CHECK IT OUT! **2.** Solve $\begin{cases} -2x + y = 8 \\ 3x + 2y = 9 \end{cases}$ by substitution.

Student to Student

Erika Chu
Terrell High School

Solving Systems by Substitution

I always look for a variable with a coefficient of 1 or −1 when deciding which equation to solve for x or y.

For the system

$\begin{cases} 2x + y = 14 \\ -3x + 4y = -10 \end{cases}$

I would solve the first equation for y because it has a coefficient of 1.

$2x + y = 14$

$y = -2x + 14$

Then I use substitution to find the values of x and y.

$-3x + 4y = -10$

$-3x + 4(-2x + 14) = -10$

$-3x + (-8x) + 56 = -10$

$-11x + 56 = -10$

$-11x = -66$

$x = 6$

$y = -2x + 14$

$y = -2(6) + 14 = 2$

The solution is $(6, 2)$.

EXAMPLE **3** *Consumer Economics Application*

One high-speed Internet provider has a $50 setup fee and costs $30 per month. Another provider has no setup fee and costs $40 per month.

a. In how many months will both providers cost the same? What will that cost be?

Write an equation for each option. Let t represent the total amount paid and m represent the number of months.

	Total paid	is	setup fee	plus	cost per month	times	months.
Option 1	t	$=$	50	$+$	30	$\cdot$	m
Option 2	t	$=$	0	$+$	40	$\cdot$	m

Step 1 $t = 50 + 30m$ *Both equations are solved for t.*
$t = 40m$

Step 2 $50 + 30m = 40m$ *Substitute 50 + 30m for t in the second equation.*

Step 3 $\underline{\quad -30m \quad -30m \quad}$ *Solve for m. Subtract 30m from both sides to combine like terms.*
$\quad 50 \quad\quad = \quad 10m$

$\dfrac{50}{10} = \dfrac{10m}{10}$ *Divide both sides by 10.*

$5 = m$

Step 4 $t = 40m$ *Write one of the original equations.*
$\quad = 40(5)$ *Substitute 5 for m.*
$\quad = 200$

Step 5 $(5, 200)$ *Write the solution as an ordered pair.*

In 5 months, the total cost for each option will be the same—$200.

b. If you plan to cancel in 1 year, which is the cheaper provider? Explain.

Option 1: $t = 50 + 30(12) = 410$ Option 2: $t = 40(12) = 480$

Option 1 is cheaper.

CHECK IT OUT!
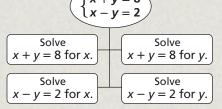
3. One cable television provider has a $60 setup fee and $80 per month, and the second has a $160 equipment fee and $70 per month.

a. In how many months will the cost be the same? What will that cost be?

b. If you plan to move in 6 months, which is the cheaper option? Explain.

THINK AND DISCUSS

1. If you graphed the equations in Example 1A, where would the lines intersect?

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, solve the system by substitution using the first step given. Show that each method gives the same solution.

$\begin{cases} x + y = 8 \\ x - y = 2 \end{cases}$

Solve $x + y = 8$ for x.	Solve $x + y = 8$ for y.
Solve $x - y = 2$ for x.	Solve $x - y = 2$ for y.

Know it! Note

6-2 **Exercises**

go.hrw.com
Homework Help Online
KEYWORD: MA7 6-2
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

Solve each system by substitution.

1. $\begin{cases} y = 5x - 10 \\ y = 3x + 8 \end{cases}$

2. $\begin{cases} 3x + y = 2 \\ 4x + y = 20 \end{cases}$

3. $\begin{cases} y = x + 5 \\ 4x + y = 20 \end{cases}$

4. $\begin{cases} x - 2y = 10 \\ \frac{1}{2}x - 2y = 4 \end{cases}$

5. $\begin{cases} y - 4x = 3 \\ 2x - 3y = 21 \end{cases}$

6. $\begin{cases} x = y - 8 \\ -x - y = 0 \end{cases}$

7. Consumer Economics The Strauss family is deciding between two lawn-care services. Green Lawn charges a $49 startup fee, plus $29 per month. Grass Team charges a $25 startup fee, plus $37 per month.

  a. In how many months will both lawn-care services cost the same? What will that cost be?

  b. If the family will use the service for only 6 months, which is the better option? Explain.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
8–10	1
11–16	2
17	3

Extra Practice
Skills Practice p. S14
Application Practice p. S33

Solve each system by substitution.

8. $\begin{cases} y = x + 3 \\ y = 2x + 4 \end{cases}$

9. $\begin{cases} y = 2x + 10 \\ y = -2x - 6 \end{cases}$

10. $\begin{cases} x + 2y = 8 \\ x + 3y = 12 \end{cases}$

11. $\begin{cases} 2x + 2y = 2 \\ -4x + 4y = 12 \end{cases}$

12. $\begin{cases} y = 0.5x + 2 \\ -y = -2x + 4 \end{cases}$

13. $\begin{cases} -x + y = 4 \\ 3x - 2y = -7 \end{cases}$

14. $\begin{cases} 3x + y = -8 \\ -2x - y = 6 \end{cases}$

15. $\begin{cases} x + 2y = -1 \\ 4x - 4y = 20 \end{cases}$

16. $\begin{cases} 4x = y - 1 \\ 6x - 2y = -3 \end{cases}$

17. Recreation Casey wants to buy a gym membership. One gym has a $150 joining fee and costs $35 per month. Another gym has no joining fee and costs $60 per month.

  a. In how many months will both gym memberships cost the same? What will that cost be?

  b. If Casey plans to cancel in 5 months, which is the better option for him? Explain.

Solve each system by substitution. Check your answer.

18. $\begin{cases} x = 5 \\ x + y = 8 \end{cases}$

19. $\begin{cases} y = -3x + 4 \\ x = 2y + 6 \end{cases}$

20. $\begin{cases} 3x - y = 11 \\ 5y - 7x = 1 \end{cases}$

21. $\begin{cases} \frac{1}{2}x + \frac{1}{3}y = 6 \\ x - y = 2 \end{cases}$

22. $\begin{cases} x = 7 - 2y \\ 2x + y = 5 \end{cases}$

23. $\begin{cases} y = 1.2x - 4 \\ 2.2x + 5 = y \end{cases}$

24. The sum of two numbers is 50. The first number is 43 less than twice the second number. Write and solve a system of equations to find the two numbers.

25. Money A jar contains n nickels and d dimes. There are 20 coins in the jar, and the total value of the coins is $1.40. How many nickels and how many dimes are in the jar? (*Hint:* Nickels are worth $0.05 and dimes are worth $0.10.)

26. Multi-Step Use the receipts below to write and solve a system of equations to find the cost of a large popcorn and the cost of a small drink.

CINEMA SNAKSHAK
Customer #3598
3 large popcorn buckets
2 small drinks
Total due: $21.00
CUSTOMER COPY

CINEMA SNAKSHAK
Customer #3599
2 large popcorn buckets
4 small drinks
Total due: $22.00
CUSTOMER COPY

27. Finance Helene invested a total of $1000 in two simple-interest bank accounts. One account paid 5% annual interest; the other paid 6% annual interest. The total amount of interest she earned after one year was $58. Write and solve a system of equations to find the amount invested in each account. (*Hint:* Change the interest rates into decimals first.)

Geometry Two angles whose measures have a sum of 90° are called complementary angles. For Exercises 28–30, x and y represent complementary angles. Find the measure of each angle.

28. $\begin{cases} x + y = 90 \\ y = 4x - 10 \end{cases}$

29. $\begin{cases} x = 2y \\ x + y = 90 \end{cases}$

30. $\begin{cases} y = 2(x - 15) \\ x + y = 90 \end{cases}$

31. Aviation With a headwind, a small plane can fly 240 miles in 3 hours. With a tailwind, the plane can fly the same distance in 2 hours. Follow the steps below to find the rates of the plane and wind.

a. Copy and complete the table. Let p be the rate of the plane and w be the rate of the wind.

	Rate	•	Time	=	Distance
With Headwind	$p - w$	•	▨	=	240
With Tailwind	▨	•	2	=	▨

b. Use the information in each row to write a system of equations.

c. Solve the system of equations to find the rates of the plane and wind.

32. Write About It Explain how to solve a system of equations by substitution.

33. Critical Thinking Explain the connection between the solution of a system solved by graphing and the solution to the same system solved by substitution.

MULTI-STEP TEST PREP

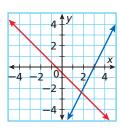**34.** This problem will prepare you for the Multi-Step Test Prep on page 412.

At the school store, Juanita bought 2 books and a backpack for a total of $26 before tax. Each book cost $8 less than the backpack.

a. Write a system of equations that can be used to find the price of each book and the price of the backpack.

b. Solve this system by substitution.

c. Solve this system by graphing. Discuss advantages and disadvantages of solving by substitution and solving by graphing.

35. Estimation Use the graph to estimate the solution to

$$\begin{cases} 2x - y = 6 \\ x + y = -0.6 \end{cases}.$$ Round your answer to the nearest tenth.

Then solve the system by substitution.

36. Elizabeth met 24 of her cousins at a family reunion. The number of male cousins m was 6 less than twice the number of female cousins f. Which system can be used to find the number of male cousins and female cousins?

(A) $\begin{cases} m + f = 24 \\ f = 2m - 6 \end{cases}$ (B) $\begin{cases} m + f = 24 \\ f = 2m \end{cases}$ (C) $\begin{cases} m = 24 + f \\ m = f - 6 \end{cases}$ (D) $\begin{cases} f = 24 - m \\ m = 2f - 6 \end{cases}$

37. Which problem is best represented by the following system $\begin{cases} d = n + 5 \\ d + n = 12 \end{cases}$?

  (F) Roger has 12 coins in dimes and nickels. There are 5 more dimes than nickels.
  (G) Roger has 5 coins in dimes and nickels. There are 12 more dimes than nickels.
  (H) Roger has 12 coins in dimes and nickels. There are 5 more nickels than dimes.
  (J) Roger has 5 coins in dimes and nickels. There are 12 more nickels than dimes.

CHALLENGE AND EXTEND

38. A car dealership has 378 cars on its lot. The ratio of new cars to used cars is 5:4. Write and solve a system of equations to find the number of new and used cars on the lot.

Solve each system by substitution.
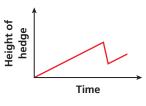
39. $\begin{cases} 2r - 3s - t = 12 \\ s + 3t = 10 \\ t = 4 \end{cases}$

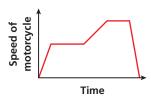**40.** $\begin{cases} x + y + z = 7 \\ y + z = 5 \\ 2y - 4z = -14 \end{cases}$

41. $\begin{cases} a + 2b + c = 19 \\ -b + c = -5 \\ 3b + 2c = 15 \end{cases}$

SPIRAL REVIEW

Write a possible situation for the given graph. *(Lesson 4-1)*

42.

43.

44.

Find the x- and y-intercepts. *(Lesson 5-2)*

45. $6x - 2y = 12$

46. $-3y + x = 15$

47. $4y - 40 = -5x$

Tell whether each ordered pair is a solution of the given system. *(Lesson 6-1)*

48. $(3, 0);\ \begin{cases} 2x - y = -6 \\ x + y = 3 \end{cases}$

49. $(-1, 4);\ \begin{cases} y - 2x = 6 \\ x + 4y = 15 \end{cases}$

50. $(5, 6);\ \begin{cases} \frac{1}{3}y + x = 7 \\ 2x = 12 \end{cases}$

Solving Systems by Elimination

Objectives

Solve systems of linear equations in two variables by elimination.

Compare and choose an appropriate method for solving systems of linear equations.
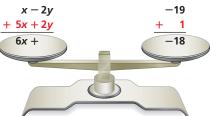
Why learn this?

You can solve a system of linear equations to determine how many flowers of each type you can buy to make a bouquet. (See Example 4.)

Another method for solving systems of equations is *elimination*. Like substitution, the goal of elimination is to get one equation that has only one variable. To do this by elimination, you add the two equations in the system together.

Remember that an equation stays balanced if you add equal amounts to both sides. So if $5x + 2y = 1$, you can add $5x + 2y$ to one side of an equation and 1 to the other side and the balance is maintained.

$$\begin{array}{ll} x - 2y & -19 \\ + 5x + 2y & + \quad 1 \\ \hline 6x + & -18 \end{array}$$

Since $-2y$ and $2y$ have **opposite coefficients**, the y-term is eliminated. The result is one equation that has only one variable: $6x = -18$.

When you use the elimination method to solve a system of linear equations, align all like terms in the equations. Then determine whether any like terms can be eliminated because they have opposite coefficients.

Solving Systems of Equations by Elimination
Step 1 Write the system so that like terms are aligned.
Step 2 Eliminate one of the variables and solve for the other variable.
Step 3 Substitute the value of the variable into one of the original equations and solve for the other variable.
Step 4 Write the answers from Steps 2 and 3 as an ordered pair, (x, y), and check.

Later in this lesson you will learn how to multiply one or more equations by a number in order to produce opposites that can be eliminated.

EXAMPLE **1** **Elimination Using Addition**

Solve $\begin{cases} x - 2y = -19 \\ 5x + 2y = 1 \end{cases}$ by elimination.

Step 1 $x - 2y = -19$ *Write the system so that like terms are aligned.*

$\underline{+ \; 5x + 2y = \qquad 1}$

Step 2 $6x \;\; + 0 = -18$ *Add the equations to eliminate the y-terms.*

$6x = -18$ *Simplify and solve for x.*

$\dfrac{6x}{6} = \dfrac{-18}{6}$ *Divide both sides by 6.*

$x = -3$

Step 3 $\; x - 2y = -19$ *Write one of the original equations.*

$-3 - 2y = -19$ *Substitute −3 for x.*

$\underline{+3 \qquad\qquad +3}$ *Add 3 to both sides.*

$-2y = -16$

$\dfrac{-2y}{-2} = \dfrac{-16}{-2}$ *Divide both sides by −2.*

$y = 8$

Step 4 $\left(-3, 8\right)$ *Write the solution as an ordered pair.*

Helpful Hint

Check your answer.

$\begin{array}{c|c} x - 2y = -19 \\ \hline -3 - 2(8) & -19 \\ -3 - 16 & -19 \\ -19 & -19 \checkmark \end{array}$

$\begin{array}{c|c} 5x + 2y = 1 \\ \hline 5(-3) + 2(8) & 1 \\ -15 + 16 & 1 \\ 1 & 1 \checkmark \end{array}$

CHECK IT OUT!   **1.** Solve $\begin{cases} y + 3x = -2 \\ 2y - 3x = 14 \end{cases}$ by elimination.

When two equations each contain the same term, you can subtract one equation from the other to solve the system. To subtract an equation, add the opposite of *each* term.

EXAMPLE **2** **Elimination Using Subtraction**

Solve $\begin{cases} 3x + 4y = 18 \\ -2x + 4y = 8 \end{cases}$ by elimination.

Step 1 $3x + 4y = 18$

$\boxed{- \left(-2x + 4y = \;\; 8\right)}$

$3x + 4y = \;\; 18$ *Add the opposite of each term*
$\underline{+ \, 2x - 4y = -8}$ *in the second equation.*

Step 2 $5x + \;\; 0 = 10$ *Eliminate the y-term.*

$5x = 10$ *Simplify and solve for x.*

$x = 2$

Step 3 $-2x + 4y = \;\; 8$ *Write one of the original equations.*

$-2(2) + 4y = \;\; 8$ *Substitute 2 for x.*

$-4 + 4y = \;\; 8$

$\underline{+4 \qquad\quad +4}$ *Add 4 to both sides.*

$4y = \; 12$ *Simplify and solve for y.*

$y = 3$

Step 4 $\left(2, 3\right)$ *Write the solution as an ordered pair.*

Remember!

Remember to check by substituting your answer into both original equations.

CHECK IT OUT!

2. Solve $\begin{cases} 3x + 3y = 15 \\ -2x + 3y = -5 \end{cases}$ by elimination.

In some cases, you will first need to multiply one or both of the equations by a number so that one variable has opposite coefficients. This will be the new Step 1.

EXAMPLE 3 | **Elimination Using Multiplication First**

Solve each system by elimination.

A $\begin{cases} 2x + y = 3 \\ -x + 3y = -12 \end{cases}$

Helpful Hint

In Example 3A, you could have also multiplied the first equation by −3 to eliminate the *y*-term.

Step 1
$$2x + y = 3$$
$$+ \, 2(-x + 3y = -12)$$
$$2x + y = 3$$
$$+ (-2x + 6y = -24)$$

Multiply each term in the second equation by 2 to get opposite x-coefficients.

Add the new equation to the first equation.

Step 2
$$7y = -21$$
$$y = -3$$

Simplify and solve for y.

Step 3 $2x + y = 3$ *Write one of the original equations.*
$2x - 3 = 3$ *Substitute −3 for y.*
$\underline{+3 \quad +3}$ *Add 3 to both sides.*
$2x = 6$ *Simplify and solve for x.*
$x = 3$

Step 4 $(3, -3)$ *Write the solution as an ordered pair.*

B $\begin{cases} 7x - 12y = -22 \\ 5x - 8y = -14 \end{cases}$

Helpful Hint

Use the techniques for finding a common denominator when trying to find values to multiply each equation by.

Step 1
$$2(7x - 12y = -22)$$
$$+ \, (-3)(5x - 8y = -14)$$
$$14x - 24y = -44$$
$$+ (-15x + 24y = 42)$$

Multiply the first equation by 2 and the second equation by −3 to get opposite y-coefficients.

Add the new equations to the first equation.

Step 2
$$-x + 0 = -2$$
$$x = 2$$

Simplify and solve for x.

Step 3 $7x - 12y = -22$ *Write one of the original equations.*
$7(2) - 12y = -22$ *Substitute 2 for x.*
$14 - 12y = -22$
$\underline{-14 \qquad -14}$ *Subtract 14 from both sides.*
$-12y = -36$ *Simplify and solve for y.*
$y = 3$

Step 4 $(2, 3)$ *Write the solution as an ordered pair.*

CHECK IT OUT! Solve each system by elimination.
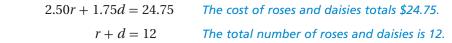
3a. $\begin{cases} 3x + 2y = 6 \\ -x + y = -2 \end{cases}$

3b. $\begin{cases} 2x + 5y = 26 \\ -3x - 4y = -25 \end{cases}$

EXAMPLE **4** *Consumer Economics Application*

Sam spent $24.75 to buy 12 flowers for his mother. The bouquet contained roses and daisies. How many of each type of flower did Sam buy?

Write a system. Use r for the number of roses and d for the number of daisies.

$$2.50r + 1.75d = 24.75 \qquad \textit{The cost of roses and daisies totals \$24.75.}$$

$$r + d = 12 \qquad \textit{The total number of roses and daisies is 12.}$$

Step 1
$$2.50r + 1.75d = 24.75$$
$$+ (-2.50)(r + d = 12) \qquad \textit{Multiply the second equation}$$
$$\textit{by } -2.50 \textit{ to get opposite}$$
$$2.50r + 1.75d = 24.75 \qquad \textit{r-coefficients.}$$
$$+ (-2.50r - 2.50d = -30.00) \qquad \textit{Add this equation to the first}$$
$$\textit{equation to eliminate the r-term.}$$

Step 2
$$-0.75d = -5.25$$
$$d = 7 \qquad \textit{Simplify and solve for d.}$$

Step 3
$$r + d = 12 \qquad \textit{Write one of the original equations.}$$
$$r + 7 = 12 \qquad \textit{Substitute 7 for d.}$$
$$\underline{-7 \quad -7} \qquad \textit{Subtract 7 from both sides.}$$
$$r = 5$$

Step 4
$$(5, 7) \qquad \textit{Write the solution as an ordered pair.}$$

Sam can buy 5 roses and 7 daisies.

4. What if...? Sally spent $14.85 to buy 13 flowers. She bought lilies, which cost $1.25 each, and tulips, which cost $0.90 each. How many of each flower did Sally buy?

All systems can be solved in more than one way. For some systems, some methods may be better than others.

Know it! Note

Systems of Linear Equations

METHOD	USE WHEN...	EXAMPLE
Graphing	• Both equations are solved for y. • You want to estimate a solution.	$\begin{cases} y = 3x + 2 \\ y = -2x + 6 \end{cases}$
Substitution	• A variable in either equation has a coefficient of 1 or -1. • Both equations are solved for the same variable. • Either equation is solved for a variable.	$\begin{cases} x + 2y = 7 \\ x = 10 - 5y \end{cases}$ or $\begin{cases} x = 2y + 10 \\ x = 3y + 5 \end{cases}$
Elimination	• Both equations have the same variable with the same or opposite coefficients. • A variable term in one equation is a multiple of the corresponding variable term in the other equation.	$\begin{cases} 3x + 2y = 8 \\ 5x + 2y = 12 \end{cases}$ or $\begin{cases} 6x + 5y = 10 \\ 3x + 2y = 15 \end{cases}$

THINK AND DISCUSS

1. Explain how multiplying the second equation in a system by -1 and eliminating by adding is the same as elimination by subtraction. Give an example of a system for which this applies.

2. Explain why it does not matter which variable you solve for first when solving a system by elimination.

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write an example of a system of equations that you could solve using the given method.

Solving Systems of Linear Equations
- Substitution
- Elimination using addition or subtraction
- Elimination using multiplication

6-3 Exercises

GUIDED PRACTICE

Solve each system by elimination.

SEE EXAMPLE 1
p. 398

1. $\begin{cases} -x + y = 5 \\ x - 5y = -9 \end{cases}$

2. $\begin{cases} x + y = 12 \\ x - y = 2 \end{cases}$

3. $\begin{cases} 2x + 5y = -24 \\ 3x - 5y = 14 \end{cases}$

SEE EXAMPLE 2
p. 398

4. $\begin{cases} x - 10y = 60 \\ x + 14y = 12 \end{cases}$

5. $\begin{cases} 5x + y = 0 \\ 5x + 2y = 30 \end{cases}$

6. $\begin{cases} -5x + 7y = 11 \\ -5x + 3y = 19 \end{cases}$

SEE EXAMPLE 3
p. 399

7. $\begin{cases} 2x + 3y = 12 \\ 5x - y = 13 \end{cases}$

8. $\begin{cases} -3x + 4y = 12 \\ 2x + y = -8 \end{cases}$

9. $\begin{cases} 2x + 4y = -4 \\ 3x + 5y = -3 \end{cases}$

SEE EXAMPLE 4
p. 400

10. **Consumer Economics** Each family in a neighborhood is contributing $20 worth of food to the neighborhood picnic. The Harlin family is bringing 12 packages of buns. The hamburger buns cost $2.00 per package. The hot-dog buns cost $1.50 per package. How many packages of each type of bun did they buy?

PRACTICE AND PROBLEM SOLVING

Solve each system by elimination.

11. $\begin{cases} -x + y = -1 \\ 2x - y = 0 \end{cases}$

12. $\begin{cases} -2x + y = -20 \\ 2x + y = 48 \end{cases}$

13. $\begin{cases} 3x - y = -2 \\ -2x + y = 3 \end{cases}$

14. $\begin{cases} x - y = 4 \\ x - 2y = 10 \end{cases}$

15. $\begin{cases} x + 2y = 5 \\ 3x + 2y = 17 \end{cases}$

16. $\begin{cases} 3x - 2y = -1 \\ 3x - 4y = 9 \end{cases}$

17. $\begin{cases} x - y = -3 \\ 5x + 3y = 1 \end{cases}$

18. $\begin{cases} 9x - 3y = 3 \\ 3x + 8y = -17 \end{cases}$
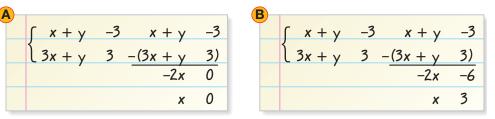
19. $\begin{cases} 5x + 2y = -1 \\ 3x + 7y = 11 \end{cases}$
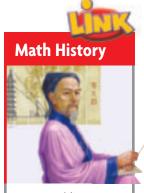
20. **Multi-Step** Mrs. Gonzalez bought centerpieces to put on each table at a graduation party. She spent $31.50. There are 8 tables each requiring either a candle or vase. Candles cost $3 and vases cost $4.25. How many of each type did she buy?

21. Geometry The difference between the length and width of a rectangle is 2 units. The perimeter is 40 units. Write and solve a system of equations to determine the length and width of the rectangle. (*Hint:* The perimeter of a rectangle is $2\ell + 2w$.)

22. /// ERROR ANALYSIS /// Which is incorrect? Explain the error.

A

$$\begin{cases} x + y & -3 & x + y & -3 \\ 3x + y & 3 & -(3x + y & 3) \\ \hline & & -2x & 0 \\ & & x & 0 \end{cases}$$

B

$$\begin{cases} x + y & -3 & x + y & -3 \\ 3x + y & 3 & -(3x + y & 3) \\ \hline & & -2x & -6 \\ & & x & 3 \end{cases}$$

23. Chemistry A chemist has a bottle of a 1% acid solution and a bottle of a 5% acid solution. She wants to mix the two solutions to get 100 mL of a 4% acid solution. Follow the steps below to find how much of each solution she should use.

	1% Solution	+	5% Solution	=	4% Solution
Amount of Solution (mL)	x	+	y	=	▪
Amount of Acid (mL)	$0.01x$	+	▪	=	$0.04(100)$

a. Copy and complete the table.

b. Use the information in the table to write a system of equations.

c. Solve the system of equations to find how much she will use from each bottle to get 100 mL of a 4% acid solution.

Critical Thinking Which method would you use to solve each system? Explain.

24. $\begin{cases} \frac{1}{2}x - 5y = 30 \\ \frac{1}{2}x + 7y = 6 \end{cases}$

25. $\begin{cases} -x + 2y = 3 \\ 4x - 5y = -3 \end{cases}$

26. $\begin{cases} 3x - y = 10 \\ 2x - y = 7 \end{cases}$

27. $\begin{cases} 3y + x = 10 \\ x = 4y + 2 \end{cases}$

28. $\begin{cases} y = -4x \\ y = 2x + 3 \end{cases}$

29. $\begin{cases} 2x + 6y = 12 \\ 4x + 5y = 15 \end{cases}$

30. Business A local boys club sold 176 bags of mulch and made a total of $520. They did not sell any of the expensive cocoa mulch. Use the table to determine how many bags of each type of mulch they sold.

Mulch Prices ($)	
Cocoa	4.75
Hardwood	3.50
Pine Bark	2.75

31. This problem will prepare you for the Multi-Step Test Prep on page 412.

a. The school store is running a promotion on school supplies. Different supplies are placed on two shelves. You can purchase 3 items from shelf A and 2 from shelf B for $16. Or you can purchase 2 items from shelf A and 3 from shelf B for $14. Write a system of equations that can be used to find the individual prices for the supplies on shelf A and on shelf B.

b. Solve the system of equations by elimination.

c. If the supplies on shelf A are normally $6 each and the supplies on shelf B are normally $3 each, how much will you save on each package plan from part **a**?

32. **Write About It** Solve the system $\begin{cases} 3x + y = 1 \\ 2x + 4y = -6 \end{cases}$. Explain how you can check your solution algebraically and graphically.

33. A math test has 25 problems. Some are worth 2 points, and some are worth 3 points. The test is worth 60 points total. Which system can be used to determine the number of 2-point problems and the number of 3-point problems on the test?

(A) $\begin{cases} x + y = 25 \\ 2x + 3y = 60 \end{cases}$ (B) $\begin{cases} x + y = 60 \\ 2x + 3y = 25 \end{cases}$ (C) $\begin{cases} x - y = 25 \\ 2x + 3y = 60 \end{cases}$ (D) $\begin{cases} x - y = 60 \\ 2x - 3y = 25 \end{cases}$

34. An electrician charges $15 plus $11 per hour. Another electrician charges $10 plus $15 per hour. For what amount of time will the cost be the same? What is that cost?

(F) 1 hour; $25

(G) $1\frac{1}{4}$ hours; $28.75

(H) $1\frac{1}{2}$ hours; $30

(J) $1\frac{3}{4}$ hours; $32.50

35. **Short Response** Three hundred and fifty-eight tickets to the school basketball game on Friday were sold. Student tickets were $1.50, and nonstudent tickets were $3.25. The school made $752.25.

   a. Write a system of linear equations that could be used to determine how many student and how many nonstudent tickets were sold. Define the variables you use.

   b. Solve the system you wrote in part **a**. How many student and how many nonstudent tickets were sold?

CHALLENGE AND EXTEND

Solve each system by any method.

36. $\begin{cases} x + 16\frac{1}{2} = -\frac{3}{4}y \\ y = \frac{1}{2}x \end{cases}$

37. $\begin{cases} 2x + y + z = 17 \\ \frac{1}{2}z = 5 \\ x - y = 5 \end{cases}$

38. $\begin{cases} x - 2y - z = -1 \\ -x + 2y + 4z = -11 \\ 2x + y + z = 1 \end{cases}$

39. The sum of the digits of a two-digit number is 5. If the number is multiplied by 3, the result is 42. Write and solve a system of equations to find the number. (*Hint:* One equation involves the digits in the number. The other equation involves the values of the digits.)

SPIRAL REVIEW

Determine a relationship between the *x*- and *y*-values. Write an equation. *(Lesson 4-3)*

40.

x	1	2	3	4
y	6	7	8	9

41.

x	1	2	3	4
y	3	6	9	12

42.

x	1	2	3	4
y	-9	-8	-7	-6

Tell whether each equation is a direct variation. If so, identify the constant of variation. *(Lesson 5-5)*

43. $x = 2y$

44. $y = -6x$

45. $y - 1 = x$

Solve each system by substitution. *(Lesson 6-2)*

46. $\begin{cases} y = x - 1 \\ x + y = 10 \end{cases}$

47. $\begin{cases} x = y - 5 \\ 2x + 1 = y \end{cases}$

48. $\begin{cases} y = 2x - 1 \\ x - y = 3 \end{cases}$

Number Theory

Solving Classic Problems

You can use systems of linear equations to solve some "classic" math problems that are common in textbooks and puzzle books.

See Skills Bank page S52

Example 1

Yuri is twice as old as Zack. Four years from now, the sum of their ages will be 23. How old is Yuri?

Step 1 Write a system. Let y represent Yuri's age. Let z represent Zack's age.

Yuri is twice as old as Zack.

$$y = 2z$$

In 4 years, the sum of their ages will be 23.

$$(y + 4) + (z + 4) = 23$$
$$y + z + 8 = 23$$
$$y + z = 15$$
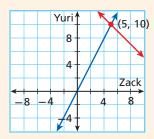
Step 2 Solve the equations for y.

$$\begin{cases} y = 2z \\ y + z = 15 \end{cases} \quad \rightarrow \quad \begin{cases} y = 2z \\ y = -z + 15 \end{cases}$$

Step 3 Graph $y = 2z$ and $y = -z + 15$.

The lines appear to intersect at $(5, 10)$.

The solution $(5, 10)$ means that Yuri is 10 years old.

Example 2

Mandy has 11 coins in dimes and quarters. The value of her coins is $2.15. How many dimes does she have?

Step 1 Write a system. Let d be the number of dimes. Let q be the number of quarters.

The total number of coins is 11.

$$q + d = 11$$

The value of the coins is $2.15.

$$0.25q + 0.10d = 2.15$$
$$100(0.25q + 0.10d = 2.15)$$
$$25q + 10d = 215$$
$$5q + 2d = 43$$

Step 2 Solve the first equation for d.

$$d = 11 - q$$

Step 3 Substitute $11 - q$ for d in the second equation.

$$5q + 2(11 - q) = 43$$
$$3q + 22 = 43 \qquad \textit{Distribute 2 and then combine like terms.}$$
$$3q = 21$$
$$q = 7 \qquad \textit{Solve for q.}$$

Step 4 Substitute 7 for q in one of the original equations.

$$q + d = 11$$
$$7 + d = 11$$
$$d = 4$$

The solution $(4, 7)$ means that there are **4** dimes and **7** quarters. Mandy has 4 dimes.

Example 3

When the digits of a two-digit number are reversed, the new number is 45 less than the original number. The sum of the digits is 7. What is the original number?

Step 1 Write expressions for the original number and the new number. Let a represent the tens digit. Let b represent the ones digit.

The original number: The new number:

$$10a + b \qquad\qquad\qquad 10b + a$$

Step 2 Write a system.

The new number is 45 less than the original number.   The sum of the digits is 7.

$$10b + a = (10a + b) - 45 \qquad a + b = 7$$
$$9b - 9a = -45$$
$$b - a = -5 \;\longrightarrow\; a - b = 5$$

Step 3 Add the equations to eliminate the b-term. Solve for a.

$$\begin{aligned} a - b &= \;\;5 \\ + (a + b &= \;\;7) \\ \hline 2a \quad\;\; &= 12 \\ a &= 6 \end{aligned}$$

Step 4 Substitute 6 for a in one of the original equations.

$$a + b = 7$$
$$6 + b = 7$$
$$b = 1$$

The solution is $(6, 1)$. This means that **6** is the tens digit and **1** is the ones digit. The original number is 61.

Check   Check your solution using the original problem.
The sum of the digits is 7: $6 + 1 = 7$ ✓
When the digits are reversed, the new number is 45 less than the original number: $16 = 61 - 45$ ✓

Try This

Solve.

1. The sum of the digits of a two-digit number is 17. When the digits are reversed, the new number is 9 more than the original number. What is the original number?

2. Vic has 14 coins in nickels and quarters. The value of his coins is $1.70. How many quarters does he have?

3. Grace is 8 years older than her brother Sam. The sum of their ages is 24. How old is Grace?

6-4 Solving Special Systems

Solve special systems of linear equations in two variables.

Classify systems of linear equations and determine the number of solutions.

Vocabulary
inconsistent system
consistent system
independent system
dependent system
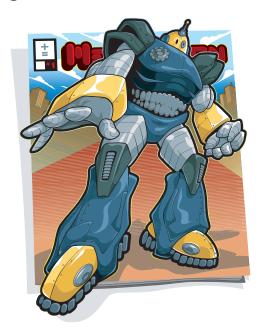
Why learn this?

Linear systems can be used to analyze business growth, such as comic book sales. (See Example 4.)

In Lesson 6-1, you saw that when two lines intersect at a point, there is exactly one solution to the system. Systems with at least one solution are called consistent .

When the two lines in a system do not intersect, they are parallel lines. There are no ordered pairs that satisfy both equations, so there is no solution. A system that has no solution is an inconsistent system .

EXAMPLE 1 Systems with No Solution

Solve $\begin{cases} y = x - 1 \\ -x + y = 2 \end{cases}$.

Method 1 Compare slopes and *y*-intercepts.

$$y = x - 1 \rightarrow y = 1x - 1 \qquad \text{Write both equations in slope-intercept form.}$$
$$-x + y = 2 \rightarrow y = 1x + 2 \qquad \text{The lines are parallel because they have the same slope and different y-intercepts.}$$

This system has no solution so it is an inconsistent system.
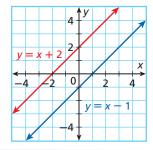
Method 2 Solve the system algebraically. Use the substitution method because the first equation is solved for *y*.

$$-x + (x - 1) = 2 \qquad \text{Substitute x − 1 for y in the second equation, and solve.}$$
$$-1 = 2 \text{✗} \qquad \text{False. The equation is a contradiction.}$$

This system has no solution so it is an inconsistent system.

Remember!

For help recalling identities and contradictions, see Lesson 2-4.

Check Graph the system to confirm that the lines are parallel.

The lines appear to be parallel.

1. Solve $\begin{cases} y = -2x + 5 \\ 2x + y = 1 \end{cases}$.

If two linear equations in a system have the same graph, the graphs are coincident lines, or the same line. There are infinitely many solutions of the system because every point on the line represents a solution of both equations.

EXAMPLE 2 **Systems with Infinitely Many Solutions**

Solve $\begin{cases} y = 2x + 1 \\ 2x - y + 1 = 0 \end{cases}$.

Method 1 Compare slopes and y-intercepts.

$y = 2x + 1 \rightarrow y = \mathbf{2}x + \mathbf{1}$ *Write both equations in slope-intercept*
$2x - y + 1 = 0 \rightarrow y = \mathbf{2}x + \mathbf{1}$ *form. The lines have the same slope and the same y-intercept.*

If this system were graphed, the graphs would be the same line. There are infinitely many solutions.

Method 2 Solve the system algebraically. Use the elimination method.

$y = 2x + 1 \rightarrow -2x + y = 1$ *Write equations to line up like terms.*
$2x - y + 1 = 0 \rightarrow \underline{\quad 2x - y = -1}$ *Add the equations.*
$0 = 0$ ✓   *True. The equation is an identity.*

There are infinitely many solutions.

> **Caution!**
> $0 = 0$ is a true statement. It does not mean the system has zero solutions or no solution.

CHECK IT OUT! **2.** Solve $\begin{cases} y = x - 3 \\ x - y - 3 = 0 \end{cases}$.

Consistent systems can either be independent or dependent.
- An **independent system** has exactly one solution. The graph of an independent system consists of two intersecting lines.
- A **dependent system** has infinitely many solutions. The graph of a dependent system consists of two coincident lines.

Know it! Note

Classification of Systems of Linear Equations

CLASSIFICATION	CONSISTENT AND INDEPENDENT	CONSISTENT AND DEPENDENT	INCONSISTENT
Number of Solutions	Exactly one	Infinitely many	None
Description	Different slopes	Same slope, same y-intercept	Same slope, different y-intercepts
Graph	Intersecting lines	Coincident lines	Parallel lines

EXAMPLE **3** **Classifying Systems of Linear Equations**

Classify each system. Give the number of solutions.

A $\begin{cases} 2y = x + 2 \\ -\dfrac{1}{2}x + y = 1 \end{cases}$

$2y = x + 2 \rightarrow y = \dfrac{1}{2}x + 1$ *Write both equations in slope-intercept form.*

$-\dfrac{1}{2}x + y = 1 \rightarrow y = \dfrac{1}{2}x + 1$ *The lines have the same slope and the same y-intercepts. They are the same.*

The system is consistent and dependent. It has infinitely many solutions.

B $\begin{cases} y = 2(x - 1) \\ y = x + 1 \end{cases}$

$y = 2(x - 1) \rightarrow y = 2x - 2$ *Write both equations in slope-intercept form.*
$y = x + 1 \rightarrow y = 1x + 1$ *The lines have different slopes. They intersect.*

The system is consistent and independent. It has one solution.

CHECK IT OUT! Classify each system. Give the number of solutions.

3a. $\begin{cases} x + 2y = -4 \\ -2(y + 2) = x \end{cases}$ **3b.** $\begin{cases} y = -2(x - 1) \\ y = -x + 3 \end{cases}$ **3c.** $\begin{cases} 2x - 3y = 6 \\ y = \dfrac{2}{3}x \end{cases}$

EXAMPLE **4** *Business Application*

The sales manager at Comics Now is comparing its sales with the sales of its competitor, Dynamo Comics. If the sales patterns continue, will the sales for Comics Now ever equal the sales for Dynamo Comics? Explain.

Comic Books Sold per Year (thousands)	2005	2006	2007	2008
Comics Now	130	170	210	250
Dynamo Comics	180	220	260	300

Use the table to write a system of linear equations. Let y represent the sales total and x represent the increase in sales.

Helpful Hint

The increase in sales is the difference between sales each year.

	Sales total	equals	increase in sales per year	times	years	plus	beginning sales.
Comics Now	y	$=$	40	$\cdot$	x	$+$	130
Dynamo Comics	y	$=$	40	$\cdot$	x	$+$	180

$\begin{cases} y = 40x + 130 \\ y = 40x + 180 \end{cases}$

$y = 40x + 130$ *Both equations are in slope-intercept form.*
$y = 40x + 180$ *The lines have the same slope, but different y-intercepts.*

The graphs of the two equations are parallel lines, so there is no solution. If the patterns continue, sales for the two companies will never be equal.

CHECK IT OUT! **4.** Matt has $100 in a checking account and deposits $20 per month. Ben has $80 in a checking account and deposits $30 per month. Will the accounts ever have the same balance? Explain.

6-4

Exercises

GUIDED PRACTICE

1. **Vocabulary** A _____?_____ system can be independent or dependent. (*consistent* or *inconsistent*)

Solve each system of linear equations.

SEE EXAMPLE **1**
p. 406

2. $\begin{cases} y = x + 1 \\ -x + y = 3 \end{cases}$

3. $\begin{cases} 3x + y = 6 \\ y = -3x + 2 \end{cases}$

4. $\begin{cases} -y = 4x + 1 \\ 4x + y = 2 \end{cases}$

SEE EXAMPLE **2**
p. 407

5. $\begin{cases} y = -x + 3 \\ x + y - 3 = 0 \end{cases}$

6. $\begin{cases} y = 2x - 4 \\ 2x - y - 4 = 0 \end{cases}$

7. $\begin{cases} -7x + y = -2 \\ 7x - y = 2 \end{cases}$

SEE EXAMPLE **3**
p. 408

Classify each system. Give the number of solutions.

8. $\begin{cases} y = 2(x + 3) \\ -2y = 2x + 6 \end{cases}$

9. $\begin{cases} y = -3x - 1 \\ 3x + y = 1 \end{cases}$

10. $\begin{cases} 9y = 3x + 18 \\ \dfrac{1}{3}x - y = -2 \end{cases}$

SEE EXAMPLE **4**
p. 408

11. **Athletics** Micah walks on a treadmill at 4 miles per hour. He has walked 2 miles when Luke starts running at 6 miles per hour on the treadmill next to him. If their rates continue, will Luke's distance ever equal Micah's distance? Explain.

PRACTICE AND PROBLEM SOLVING

Solve each system of linear equations.

12. $\begin{cases} y = 2x - 2 \\ -2x + y = 1 \end{cases}$

13. $\begin{cases} x + y = 3 \\ y = -x - 1 \end{cases}$

14. $\begin{cases} x + 2y = -4 \\ y = -\dfrac{1}{2}x - 4 \end{cases}$

15. $\begin{cases} -6 + y = 2x \\ y = 2x - 36 \end{cases}$

16. $\begin{cases} y = -2x + 3 \\ 2x + y - 3 = 0 \end{cases}$

17. $\begin{cases} y = x - 2 \\ x - y - 2 = 0 \end{cases}$

18. $\begin{cases} x + y = -4 \\ y = -x - 4 \end{cases}$

19. $\begin{cases} -9x - 3y = -18 \\ 3x + y = 6 \end{cases}$

Classify each system. Give the number of solutions.

20. $\begin{cases} y = -x + 5 \\ x + y = 5 \end{cases}$

21. $\begin{cases} y = -3x + 2 \\ y = 3x \end{cases}$

22. $\begin{cases} y - 1 = 2x \\ y = 2x - 1 \end{cases}$

23. **Sports** Mandy is skating at 5 miles per hour. Nikki is skating at 6 miles per hour and started 1 mile behind Mandy. If their rates stay the same, will Mandy catch up with Nikki? Explain.

24. **Multi-Step** Photocopier A can print 35 copies per minute. Photocopier B can print 35 copies per minute. Copier B is started and makes 10 copies. Copier A is then started. If the copiers continue, will the number of copies from machine A ever equal the number of copies from machine B? Explain.

25. **Entertainment** One week Trey rented 4 DVDs and 2 video games for $18. The next week he rented 2 DVDs and 1 video game for $9. Find the rental costs for each video game and DVD. Explain your answer.

26. Rosa bought 1 pound of cashews and 2 pounds of peanuts for $10. At the same store, Sabrina bought 2 pounds of cashews and 1 pound of peanuts for $11. Find the cost per pound for cashews and peanuts.

27. **Geology** Pam and Tommy collect geodes. Pam's parents gave her 2 geodes to start her collection, and she buys 4 every year. Tommy has 2 geodes that were given to him for his birthday. He buys 4 every year. If Pam and Tommy continue to buy the same amount of geodes per year, when will Tommy have as many geodes as Pam? Explain your answer.

28. Use the data given in the tables.

x	3	4	5	6
y	6	8	10	12

x	12	13	14	15
y	24	26	28	30

   a. Write an equation to describe the data in each table.

   b. Graph the system of equations from part **a.** Describe the graph.

   c. How could you have predicted the graph by looking at the equations?

   d. **What if...?** Each y-value in the second table increases by 1. How does this affect the graphs of the two equations? How can you tell how the graphs would be affected without actually graphing?

29. **Critical Thinking** Describe the graphs of two equations if the result of solving the system by substitution or elimination is the statement $1 = 3$.

Geology

Geodes are rounded, hollow rock formations. Most are partially or completely filled with layers of colored quartz crystals. The world's largest geode was discovered in Spain in 2000. It is 26 feet long and 5.6 feet high.
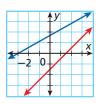
MULTI-STEP TEST PREP

30. This problem will prepare you for the Multi-Step Test Prep on page 412.

   The Crusader pep club is selling team buttons that support the sports teams. They contacted Buttons, Etc. which charges $50 plus $1.10 per button, and Logos, which charges $40 plus $1.10 per button.

   a. Write an equation for each company's cost.

   b. Use the system from part **a** to find when the price for both companies is the same. Explain.

   c. What part of the equation should the pep club negotiate to change so that the cost of Buttons, Etc. is the same as Logos? What part of the equation should change in order to get a better price?

31. **///ERROR ANALYSIS///** Student A says there is no solution to the graphed system of equations. Student B says there is one solution. Which student is incorrect? Explain the error.

32. Write About It Compare the graph of a system that is consistent and independent with the graph of a system that is consistent and dependent.

33. Which of the following classifications fit the following system?

$$\begin{cases} 2x - y = 3 \\ 6x - 3y = 9 \end{cases}$$

(A) Inconsistent and independent   (C) Inconsistent and dependent

(B) Consistent and independent   (D) Consistent and dependent

34. Which of the following would be enough information to classify a system of two linear equations?

(F) The graphs have the same slope.

(G) The y-intercepts are the same.

(H) The graphs have different slopes.

(J) The y-intercepts are different.

CHALLENGE AND EXTEND

35. What conditions are necessary for the system $\begin{cases} y = 2x + p \\ y = 2x + q \end{cases}$ to have infinitely many solutions? no solution?

36. Solve the systems in parts **a** and **b.** Use this information to make a conjecture about all solutions that exist for the system in part **c.**

a. $\begin{cases} 3x + 4y = 0 \\ 4x + 3y = 0 \end{cases}$ **b.** $\begin{cases} 2x + 5y = 0 \\ 5x + 2y = 0 \end{cases}$ **c.** $\begin{cases} ax + by = 0 \\ bx + ay = 0 \end{cases}$, for $a > 0, b > 0, a \neq b$

SPIRAL REVIEW

Use the map to find the actual distances between each pair of cities. *(Lesson 2-6)*

37. from Hon to Averly

38. from Averly to Lewers

Determine if each sequence appears to be an arithmetic sequence. If so, find the common difference and the next three terms. *(Lesson 4-6)*

39. 1, 3, 5, 9, …   **40.** 1, 5, 9, 13, …   **41.** $0, -1\frac{1}{2}, -3, -4\frac{1}{2}, …$

Solve each system by graphing. *(Lesson 6-1)*

42. $\begin{cases} y = x - 2 \\ y = -x + 4 \end{cases}$ **43.** $\begin{cases} y = 2x \\ x + y = -6 \end{cases}$ **44.** $\begin{cases} y = -\frac{1}{2}x \\ y - x = 9 \end{cases}$
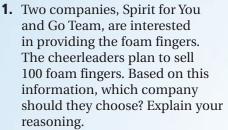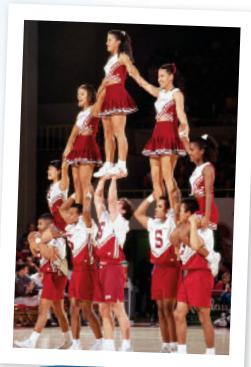
Systems of Equations

We've Got Spirit Some cheerleaders are going to sell spirit bracelets and foam fingers to raise money for traveling to away games.

1. Two companies, Spirit for You and Go Team, are interested in providing the foam fingers. The cheerleaders plan to sell 100 foam fingers. Based on this information, which company should they choose? Explain your reasoning.

Company	Design fee	Cost per item
Spirit for You	$35	$2.50
Go Team	$20	$3.00

2. The cheerleaders sold foam fingers for $5 and spirit bracelets for $4. They sold 40 more foam fingers than bracelets, and they earned $965. Write a system of equations to describe this situation.

3. Solve this system using at least two different methods. Explain each method.

4. Using the company you chose in Problem 1, how much profit did the cheerleaders make from the foam fingers alone? (*Hint:* profit = amount earned − expenses)

5. What is the maximum price the cheerleaders could pay for each spirit bracelet in order to make a total profit of $500?

READY TO GO ON?

Quiz for Lessons 6-1 Through 6-4

6-1 Solving Systems by Graphing

Tell whether the ordered pair is a solution of the given system.

1. $(-2, 1); \begin{cases} y = -2x - 3 \\ y = x + 3 \end{cases}$

2. $(9, 2); \begin{cases} x - 4y = 1 \\ 2x - 3y = 3 \end{cases}$

3. $(3, -1); \begin{cases} y = -\frac{1}{3}x \\ y + 2x = 5 \end{cases}$

Solve each system by graphing.

4. $\begin{cases} y = x + 5 \\ y = \frac{1}{2}x + 4 \end{cases}$

5. $\begin{cases} y = -x - 2 \\ 2x - y = 2 \end{cases}$

6. $\begin{cases} \frac{2}{3}x + y = -3 \\ 4x + y = 7 \end{cases}$

7. Banking Christiana and Marlena opened their first savings accounts on the same day. Christiana opened her account with $50 and plans to deposit $10 every month. Marlena opened her account with $30 and plans to deposit $15 every month. After how many months will their two accounts have the same amount of money? What will that amount be?

6-2 Solving Systems by Substitution

Solve each system by substitution.

8. $\begin{cases} y = -x + 5 \\ 2x + y = 11 \end{cases}$

9. $\begin{cases} 4x - 3y = -1 \\ 3x - y = -2 \end{cases}$

10. $\begin{cases} y = -x \\ y = -2x - 5 \end{cases}$

6-3 Solving Systems by Elimination

Solve each system by elimination.

11. $\begin{cases} x + 3y = 15 \\ 2x - 3y = -6 \end{cases}$

12. $\begin{cases} x + y = 2 \\ 2x + y = -1 \end{cases}$

13. $\begin{cases} -2x + 5y = -1 \\ 3x + 2y = 11 \end{cases}$

14. It takes Akira 10 minutes to make a black and white drawing and 25 minutes for a color drawing. On Saturday he made a total of 9 drawings in 2 hours. Write and solve a system of equations to determine how many drawings of each type Akira made.

6-4 Solving Special Systems

Solve each system of linear equations.

15. $\begin{cases} y = -2x - 6 \\ 2x + y = 5 \end{cases}$

16. $\begin{cases} x + y = 2 \\ 2x + 2y = -6 \end{cases}$

17. $\begin{cases} y = -2x + 4 \\ 2x + y = 4 \end{cases}$

Classify each system. Give the number of solutions.

18. $\begin{cases} 3x = -6y + 3 \\ 2y = -x + 1 \end{cases}$

19. $\begin{cases} y = -4x + 2 \\ 4x + y = -2 \end{cases}$

20. $\begin{cases} 4x - 3y = 8 \\ y = 4(x + 2) \end{cases}$

Solving Linear Inequalities

Objective
Graph and solve linear inequalities in two variables.

Vocabulary
linear inequality
solution of a linear inequality

Who uses this?
Consumers can use linear inequalities to determine how much food they can buy for an event. (See Example 3.)

A **linear inequality** is similar to a linear equation, but the equal sign is replaced with an inequality symbol. A **solution of a linear inequality** is any ordered pair that makes the inequality true.

E X A M P L E **1** **Identifying Solutions of Inequalities**

Tell whether the ordered pair is a solution of the inequality.

A $(7, 3); y < x - 1$

$$y < x - 1$$

| 3 | 7 − 1 |

$$3 < 6 \checkmark$$

Substitute (7, 3) for (x, y).

$(7, 3)$ is a solution.

B $(4, 5); y > 3x + 2$

$$y > 3x + 2$$

| 5 | 3(4) + 2 |
| 5 | 12 + 2 |

$$5 > 14 \; ✗$$

Substitute (4, 5) for (x, y).

$(4, 5)$ is not a solution.

CHECK IT OUT! Tell whether the ordered pair is a solution of the inequality.
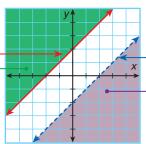
1a. $(4, 5); y < x + 1$ **1b.** $(1, 1); y > x - 7$

A linear inequality describes a region of a coordinate plane called a *half-plane*. All points in the region are solutions of the linear inequality. The boundary line of the region is the graph of the related equation.

When the inequality is written as $y \leq$ or $y \geq$, the points on the boundary line are solutions of the inequality, and the line is **solid**.

When the inequality is written as $y <$ or $y >$, the points on the boundary line are not solutions of the inequality, and the line is **dashed**.

When the inequality is written as $y >$ or $y \geq$, the points **above** the boundary line are solutions of the inequality.

When the inequality is written as $y <$ or $y \leq$, the points **below** the boundary line are solutions of the inequality.

Graphing Linear Inequalities	
Step 1	Solve the inequality for y (slope-intercept form).
Step 2	Graph the boundary line. Use a solid line for $\leq$ or $\geq$. Use a dashed line for $<$ or $>$.
Step 3	Shade the half-plane above the line for $y >$ or $y \geq$. Shade the half-plane below the line for $y <$ or $y \leq$. Check your answer.

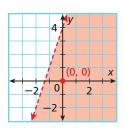**EXAMPLE 2** **Graphing Linear Inequalities in Two Variables**

Graph the solutions of each linear inequality.

A $y < 3x + 4$

Step 1 The inequality is already solved for y.
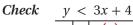
Step 2 Graph the boundary line $y = 3x + 4$. Use a dashed line for $<$.

Step 3 The inequality is $<$, so shade below the line.

Helpful Hint

The point $(0, 0)$ is a good test point to use if it does not lie on the boundary line.

Check

$$y < 3x + 4$$

0	$3(0) + 4$
0	$0 + 4$
$0 < 4$ ✓	

Substitute $(0, 0)$ for (x, y) because it is not on the boundary line.

The point $(0, 0)$ satisfies the inequality, so the graph is shaded correctly.

B $3x + 2y \geq 6$

Step 1 Solve the inequality for y.
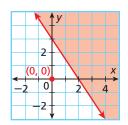
$$3x + 2y \geq 6$$
$$\underline{-3x \qquad -3x}$$
$$2y \geq -3x + 6$$
$$y \geq -\frac{3}{2}x + 3$$

Step 2 Graph the boundary line $y = -\frac{3}{2}x + 3$. Use a solid line for $\geq$.

Step 3 The inequality is $\geq$, so shade above the line.

Check

$$y \geq \frac{3}{2}x + 3$$

0	$\frac{3}{2}(0) + 3$
0	$0 + 3$
$0 \geq 3$ ✗	

A false statement means that the half-plane containing $(0, 0)$ should NOT be shaded. $(0, 0)$ is not one of the solutions, so the graph is shaded correctly.

CHECK IT OUT! **Graph the solutions of each linear inequality.**

2a. $4x - 3y > 12$ **2b.** $2x - y - 4 > 0$ **2c.** $y \geq -\frac{2}{3}x + 1$

EXAMPLE 3 *Consumer Economics Application*

Sarah can spend at most $7.50 on vegetables. Broccoli costs $1.25 per bunch and carrots cost $0.75 per package.

a. Write a linear inequality to describe the situation.

Let x represent the number of bunches of broccoli and let y represent the number of packages of carrots.

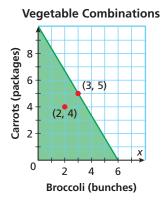Write an inequality. Use $\leq$ for "at most."

Cost of broccoli	plus	cost of carrots	is at most	$7.50.
$1.25x$	$+$	$0.75y$	$\leq$	7.50

Solve the inequality for y.

$$1.25x + 0.75y \leq 7.50$$

$$\mathbf{100}(1.25x + 0.75y) \leq \mathbf{100}(7.50)$$ *You can multiply both sides of the inequality by 100 to eliminate the decimals.*

$$125x + 75y \leq 750$$

$$\underline{\mathbf{-125x} \qquad \mathbf{-125x}}$$ *Subtract 125x from both sides.*

$$75y \leq 750 - 125x$$

$$\frac{75y}{\mathbf{75}} \leq \frac{750 - 125x}{\mathbf{75}}$$ *Divide both sides by 75.*

$$y \leq 10 - \frac{5}{3}x$$

b. Graph the solutions.

Step 1 Since Sarah cannot buy a negative amount of vegetables, the system is graphed only in Quadrant I. Graph the boundary line $y = -\frac{5}{3}x + 10$. Use a solid line for $\leq$.

Step 2 Shade below the line. Sarah must buy whole numbers of bunches or packages. All the points on or below the line with whole number coordinates are the different combinations of broccoli and carrots that Sarah can buy.

Vegetable Combinations

c. Give two combinations of vegetables that Sarah can buy.
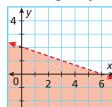
Two different combinations that Sarah could buy for $7.50 or less are 2 bunches of broccoli and 4 packages of carrots, or 3 bunches of broccoli and 5 packages of carrots.

CHECK IT OUT!

3. What if...? Dirk is going to bring two types of olives to the Honor Society induction and can spend no more than $6. Green olives cost $2 per pound and black olives cost $2.50 per pound.

a. Write a linear inequality to describe the situation.

b. Graph the solutions.

c. Give two combinations of olives that Dirk could buy.

EXAMPLE **4** **Writing an Inequality from a Graph**

Write an inequality to represent each graph.
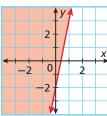
A

y-intercept: **2**; slope: $-\dfrac{1}{3}$

Write an equation in slope-intercept form.

$$y = mx + b \longrightarrow y = -\dfrac{1}{3}x + 2$$

The graph is shaded *below* a *dashed* boundary line.

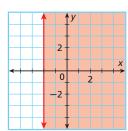Replace = with < to write the inequality $y < -\dfrac{1}{3}x + 2$.

B

y-intercept: **−2**; slope: **5**

Write an equation in slope-intercept form.

$$y = mx + b \longrightarrow y = 5x + (-2)$$

The graph is shaded *above* a *solid* boundary line.

Replace = with ≥ to write the inequality $y \geq 5x - 2$.

C

y-intercept: **none**; slope: **undefined**
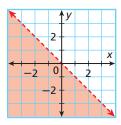
The graph is a vertical line at $x = -2$.

The graph is shaded on the *right* side of a *solid* boundary line.
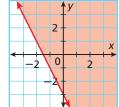
Replace = with ≥ to write the inequality $x \geq -2$.

CHECK IT OUT! **Write an inequality to represent each graph.**

4a.

4b.

THINK AND DISCUSS

1. Tell how graphing a linear inequality is the same as graphing a linear equation. Tell how it is different.

2. Explain how you would write a linear inequality from a graph.

3. GET ORGANIZED Copy and complete the graphic organizer.

Know it! *Note*

Inequality	$y < 5x + 2$	$y > 7x - 3$	$y \leq 9x + 1$	$y \geq -3x - 2$
Symbol	<			
Boundary Line	Dashed			
Shading	Below			

6-5 **Exercises**

go.hrw.com
Homework Help Online
KEYWORD: MA7 6-5
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

1. **Vocabulary** Can a *solution of a linear inequality* lie on a dashed boundary line? Explain.

SEE EXAMPLE 1
p. 414

Tell whether the ordered pair is a solution of the given inequality.

2. $(0, 3)$; $y \leq -x + 3$ 3. $(2, 0)$; $y > -2x - 2$ 4. $(-2, 1)$; $y < 2x + 4$

SEE EXAMPLE 2
p. 415

Graph the solutions of each linear inequality.

5. $y \leq -x$ 6. $y > 3x + 1$ 7. $-y < -x + 4$ 8. $-y \geq x + 1$

SEE EXAMPLE 3
p. 416

9. **Multi-Step** Jack is making punch with orange juice and pineapple juice. He can make at most 16 cups of punch.
   a. Write an inequality to describe the situation.
   b. Graph the solutions.
   c. Give two possible combinations of cups of orange juice and pineapple juice that Jack can use in his punch.

SEE EXAMPLE 4
p. 417

Write an inequality to represent each graph.

10.

11.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
12–14	1
15–18	2
19	3
20–21	4

Extra Practice
Skills Practice p. S15
Application Practice p. S33
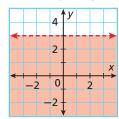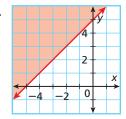
Tell whether the ordered pair is a solution of the given inequality.

12. $(2, 3)$; $y \geq 2x + 3$ 13. $(1, -1)$; $y < 3x - 3$ 14. $(0, 7)$; $y > 4x + 7$

Graph the solutions of each linear inequality.

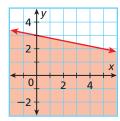15. $y > -2x + 6$ 16. $-y \geq 2x$ 17. $x + y \leq 2$ 18. $x - y \geq 0$
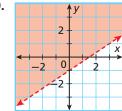
19. **Multi-Step** Beverly is serving hamburgers and hot dogs at her cookout. Hamburger meat costs $3 per pound, and hot dogs cost $2 per pound. She wants to spend no more than $30.
   a. Write an inequality to describe the situation.
   b. Graph the solutions.
   c. Give two possible combinations of pounds of hamburger and hot dogs that Beverly can buy.

Write an inequality to represent each graph.

20.

21.

22. Business An electronics store makes $125 profit on every DVD player it sells and $100 on every CD player it sells. The store owner wants to make a profit of at least $500 a day selling DVD players and CD players.

    a. Write a linear inequality to determine the number of DVD players x and the number of CD players y that the owner needs to sell to meet his goal.

    b. Graph the linear inequality.

    c. Describe the possible values of x. Describe the possible values of y.

    d. List three possible combinations of DVD players and CD players that the owner could sell to meet his goal.

Graph the solutions of each linear inequality.

23. $y \leq 2 - 3x$ **24.** $-y < 7 + x$ **25.** $2x - y \leq 4$ **26.** $3x - 2y > 6$

27. Geometry Marvin has 18 yards of fencing that he can use to put around a rectangular garden.

    a. Write a linear inequality that describes the possible lengths and widths of the garden.

    b. Graph the inequality and list three possible solutions to the problem.

    c. What are the dimensions of the largest *square* garden that can be fenced in with whole-number dimensions?

28. Hobbies Stephen wants to buy yellow tangs and clown fish for his saltwater aquarium. He wants to spend no more than $77 on fish. At the store, yellow tangs cost $15 each and clown fish cost $11 each. Write and graph a linear inequality to find the number of yellow tangs x and the number of clown fish y that Stephen could purchase. Name a solution of your inequality that is not reasonable for the situation. Explain.

Graph each inequality on a coordinate plane.

29. $y > 1$ **30.** $-2 < x$ **31.** $x \geq -3$ **32.** $y \leq 0$

33. $0 \geq x$ **34.** $-12 + y > 0$ **35.** $x + 7 < 7$ **36.** $-4 \geq x - y$

37. School At a high school football game, tickets at the gate cost $7 per adult and $4 per student. Write a linear inequality to determine the number of adult and student tickets that need to be sold so that the amount of money taken in at the gate is at least $280. Graph the inequality and list three possible solutions.

38. Critical Thinking Why must a region of a coordinate plane be shaded to show all solutions of a linear inequality?
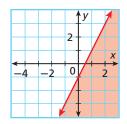
39. Write About It Give a real-world situation that can be described by a linear inequality. Then graph the inequality and give two solutions.

MULTI-STEP TEST PREP

40. This problem will prepare you for the Multi-Step Test Prep on page 428.
Gloria is making teddy bears. She is making boy and girl bears. She has enough stuffing to create 50 bears. Let x represent the number of girl bears and y represent the number of boy bears.

    a. Write an inequality that shows the possible number of boy and girl bears Jenna can make.

    b. Graph the inequality.

    c. Give three possible solutions for the numbers of boy and girl bears that can be made.

41. ///ERROR ANALYSIS/// Student A wrote $y < 2x - 1$ as the inequality represented by the graph. Student B wrote $y \le 2x - 1$ as the inequality represented by the graph. Which student is incorrect? Explain the error.

42. Write About It How do you decide to shade above or below an inequality? What does this shading represent?

43. Which point is a solution of the inequality $y > -x + 3$?

    Ⓐ $(0, 3)$ Ⓑ $(1, 4)$ Ⓒ $(-1, 4)$ Ⓓ $(0, -3)$

44. Which inequality is represented by the graph at right?

    Ⓕ $2x + y \ge 3$ Ⓗ $2x + y \le 3$

    Ⓖ $2x + y > 3$ Ⓙ $2x + y < 3$

45. Which of the following describes the graph of $3 \le x$?

    Ⓐ The boundary line is dashed, and the shading is to the right.

    Ⓑ The boundary line is dashed, and the shading is to the left.

    Ⓒ The boundary line is solid, and the shading is to the right.

    Ⓓ The boundary line is solid, and the shading is to the left.

CHALLENGE AND EXTEND

Graph each inequality.

46. $0 \ge -6 - 2x - 5y$ **47.** $y > |x|$ **48.** $y \ge |x - 3|$

49. A linear inequality has the points $(0, 3)$ and $(-3, 1.5)$ as solutions on the boundary line. Also, the point $(1, 1)$ is not a solution. Write the linear inequality.

50. Two linear inequalities are graphed on the same coordinate plane. The point $(0, 0)$ is a solution of both inequalities. The entire coordinate plane is shaded except for Quadrant I. What are the two inequalities?

SPIRAL REVIEW

Tell whether each function is linear. If so, graph the function. *(Lesson 5-1)*

51. $y = 2x - 4$ **52.** $y = x^2 + 2$ **53.** $y = 3$

Write an equation in slope-intercept form for the line through the two points. *(Lesson 5-7)*

54. $(0, 9)$ and $(5, 2)$ **55.** $(-5, -2)$ and $(7, 7)$ **56.** $(0, 0)$ and $(-8, -10)$

57. $(-1, -2)$ and $(1, 4)$ **58.** $(2, 2)$ and $(6, 5)$ **59.** $(-3, 2)$ and $(3, -1)$

Solve each system by elimination. *(Lesson 6-3)*

60. $\begin{cases} x + 6y = 14 \\ x - 6y = -10 \end{cases}$ **61.** $\begin{cases} x + y = 13 \\ 3x + y = 9 \end{cases}$ **62.** $\begin{cases} 2x - 4y = 18 \\ 5x - y = 36 \end{cases}$

63. $\begin{cases} 2y + x = 12 \\ y - 2x = 1 \end{cases}$ **64.** $\begin{cases} 2y - 6x = -8 \\ y = -5x + 12 \end{cases}$ **65.** $\begin{cases} 2x + 3y = 33 \\ y = \frac{1}{4}x \end{cases}$

Solving Systems of Linear Inequalities

Objective
Graph and solve systems of linear inequalities in two variables.

Vocabulary
system of linear inequalities
solution of a system of linear inequalities

Who uses this?
The owner of a surf shop can use systems of linear inequalities to determine how many surfboards and wakeboards need to be sold to make a certain profit. (See Example 4.)

A **system of linear inequalities** is a set of two or more linear inequalities containing two or more variables. The **solutions of a system of linear inequalities** consists of all the ordered pairs that satisfy all the linear inequalities in the system.

EXAMPLE 1 **Identifying Solutions of Systems of Linear Inequalities**

Tell whether the ordered pair is a solution of the given system.

Remember!
An ordered pair must be a solution of all inequalities to be a solution of the system.

A $(2, 1)$; $\begin{cases} y < -x + 4 \\ y \le x + 1 \end{cases}$

$(2, 1)$

$y < -x + 4$
$1 \mid -2 + 4$
$1 < 2$ ✓

$(2, 1)$

$y \le x + 1$
$1 \mid 2 + 1$
$1 \le 3$ ✓

$(2, 1)$ is a solution to the system because it satisfies both inequalities.

B $(2, 0)$; $\begin{cases} y \ge 2x \\ y < x + 1 \end{cases}$

$(2, 0)$

$y \ge 2x$
$0 \mid 2(2)$
$0 \ge 4$ ✗

$(2, 0)$

$y < x + 1$
$0 \mid 2 + 1$
$0 < 3$ ✓

$(2, 0)$ is not a solution to the system because it does not satisfy both inequalities.

CHECK IT OUT!

Tell whether the ordered pair is a solution of the given system.

1a. $(0, 1)$; $\begin{cases} y < -3x + 2 \\ y \ge x - 1 \end{cases}$

1b. $(0, 0)$; $\begin{cases} y > -x + 1 \\ y > x - 1 \end{cases}$

To show all the solutions of a system of linear inequalities, graph the solutions of each inequality. The solutions of the system are represented by the overlapping shaded regions. Below are graphs of Examples 1A and 1B.

Example 1A

(2, 1) is in the overlapping shaded regions, so it is a solution.

Example 1B

(2, 0) is not in the overlapping shaded regions, so it is not a solution.

EXAMPLE 2 **Solving a System of Linear Inequalities by Graphing**

Graph the system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

$$\begin{cases} 8x + 4y \le 12 \\ y > \dfrac{1}{2}x - 2 \end{cases}$$
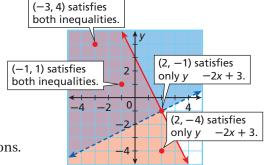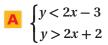
$8x + 4y \le 12$ *Write the first inequality in slope-intercept form.*
$\quad\quad 4y \le -8x + 12$
$\quad\quad\quad y \le -2x + 3$

Graph the system.

$$\begin{cases} y \le -2x + 3 \\ y > \dfrac{1}{2}x - 2 \end{cases}$$

$(-1, 1)$ and $(-3, 4)$ are solutions.
$(2, -1)$ and $(2, -4)$ are not solutions.

(−3, 4) satisfies both inequalities.

(−1, 1) satisfies both inequalities.

(2, −1) satisfies only $y \le -2x + 3$.

(2, −4) satisfies only $y \le -2x + 3$.

CHECK IT OUT! Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

2a. $\begin{cases} y \le x + 1 \\ y > 2 \end{cases}$ **2b.** $\begin{cases} y > x - 7 \\ 3x + 6y \le 12 \end{cases}$
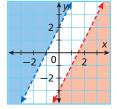
In Lesson 6-4, you saw that in systems of linear equations, if the lines are parallel, there are no solutions. With systems of linear inequalities, that is not always true.

EXAMPLE 3 **Graphing Systems with Parallel Boundary Lines**
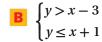
Graph each system of linear inequalities.

A $\begin{cases} y < 2x - 3 \\ y > 2x + 2 \end{cases}$ **B** $\begin{cases} y > x - 3 \\ y \le x + 1 \end{cases}$ **C** $\begin{cases} y \le -3x - 2 \\ y \le -3x + 4 \end{cases}$
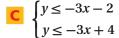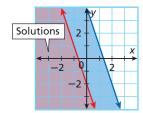
Solutions

Solutions
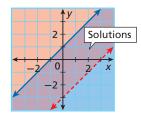
This system has no solution.

The solutions are all points between the parallel lines and on the solid line.

The solutions are the same as the solutions of $y \le -3x - 2$.

Graph each system of linear inequalities.

3a. $\begin{cases} y > x + 1 \\ y \le x - 3 \end{cases}$ **3b.** $\begin{cases} y \ge 4x - 2 \\ y \le 4x + 2 \end{cases}$ **3c.** $\begin{cases} y > -2x + 3 \\ y > -2x \end{cases}$

EXAMPLE 4 **Business Application**

A surf shop makes the profits given in the table. The shop owner sells at least 10 surfboards and at least 20 wakeboards per month. He wants to earn at least $2000 a month. Show and describe all possible combinations of surfboards and wakeboards that the store owner needs to sell to meet his goals. List two possible combinations.

Profit per Board Sold ($)	
Surfboard	150
Wakeboard	100

Step 1 Write a system of inequalities.
Let x represent the number of surfboards and y represent the number of wakeboards.

$x \ge 10$ *He sells at least 10 surfboards.*

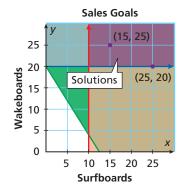$y \ge 20$ *He sells at least 20 wakeboards.*
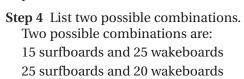
$150x + 100y \ge 2000$ *He wants to earn a total of at least $2000.*

Caution!

An ordered pair solution of the system need not have whole numbers, but answers to many application problems may be restricted to whole numbers.

Step 2 Graph the system.
The graph should be in only the first quadrant because sales are not negative.

Step 3 Describe all possible combinations.
To meet the sales goals, the shop could sell any combination represented by an ordered pair of whole numbers in the solution region. Answers must be whole numbers because the shop cannot sell part of a surfboard or wakeboard.

Step 4 List two possible combinations.
Two possible combinations are:
15 surfboards and 25 wakeboards
25 surfboards and 20 wakeboards

Sales Goals
(15, 25)
(25, 20)
Solutions

4. At her party, Alice is serving pepper jack cheese and cheddar cheese. She wants to have at least 2 pounds of each. Alice wants to spend at most $20 on cheese. Show and describe all possible combinations of the two cheeses Alice could buy. List two possible combinations.
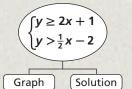
Price per Pound ($)	
Pepper Jack	4
Cheddar	2

THINK AND DISCUSS

1. How would you write a system of linear inequalities from a graph?

Know it! Note

2. GET ORGANIZED Copy and complete each part of the graphic organizer. In each box, draw a graph and list one solution.

$\begin{cases} y \ge 2x + 1 \\ y > \frac{1}{2}x - 2 \end{cases}$ $\begin{cases} y < 2x + 1 \\ y \ge \frac{1}{2}x - 2 \end{cases}$

| Graph | Solution | | Graph | Solution |

6-6

Exercises

go.hrw.com
Homework Help Online
KEYWORD: MA7 6-6
Parent Resources Online
KEYWORD: MA7 Parent

GUIDED PRACTICE

1. **Vocabulary** A solution of a system of inequalities is a solution of _____?_____ of the inequalities in the system. (*at least one* or *all*)

SEE EXAMPLE 1
p. 421

Tell whether the ordered pair is a solution of the given system.

2. $(0, 0);$ $\begin{cases} y < -x + 3 \\ y < x + 2 \end{cases}$

3. $(0, 0);$ $\begin{cases} y < 3 \\ y > x - 2 \end{cases}$

4. $(1, 0);$ $\begin{cases} y > 3x \\ y \le x + 1 \end{cases}$

SEE EXAMPLE 2
p. 422

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

5. $\begin{cases} y < 2x - 1 \\ y > 2 \end{cases}$

6. $\begin{cases} x < 3 \\ y > x - 2 \end{cases}$

7. $\begin{cases} y \ge 3x \\ 3x + y \ge 3 \end{cases}$

8. $\begin{cases} 2x - 4y \le 8 \\ y > x - 2 \end{cases}$

SEE EXAMPLE 3
p. 422

Graph each system of linear inequalities.

9. $\begin{cases} y > 2x + 3 \\ y < 2x \end{cases}$

10. $\begin{cases} y \le -3x - 1 \\ y \ge -3x + 1 \end{cases}$

11. $\begin{cases} y > 4x - 1 \\ y \le 4x + 1 \end{cases}$

12. $\begin{cases} y < -x + 3 \\ y > -x + 2 \end{cases}$

13. $\begin{cases} y > 2x - 1 \\ y > 2x - 4 \end{cases}$

14. $\begin{cases} y \le -3x + 4 \\ y \le -3x - 3 \end{cases}$

SEE EXAMPLE 4
p. 423

15. **Business** Sandy makes $2 profit on every cup of lemonade that she sells and $1 on every cupcake that she sells. Sandy wants to sell at least 5 cups of lemonade and at least 5 cupcakes per day. She wants to earn at least $25 per day. Show and describe all the possible combinations of lemonade and cupcakes that Sandy needs to sell to meet her goals. List two possible combinations.

PRACTICE AND PROBLEM SOLVING

Independent Practice	
For Exercises	See Example
16–18	1
19–22	2
23–28	3
29	4

Extra Practice
Skills Practice p. S15
Application Practice p. S33

Tell whether the ordered pair is a solution of the given system.

16. $(0, 0);$ $\begin{cases} y > -x - 1 \\ y < 2x + 4 \end{cases}$

17. $(0, 0);$ $\begin{cases} x + y < 3 \\ y > 3x - 4 \end{cases}$

18. $(1, 0);$ $\begin{cases} y > 3x \\ y > 3x + 1 \end{cases}$

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

19. $\begin{cases} y < -3x - 3 \\ y \ge 0 \end{cases}$

20. $\begin{cases} y < -1 \\ y > 2x - 1 \end{cases}$

21. $\begin{cases} y > 2x + 4 \\ 6x + 2y \ge -2 \end{cases}$

22. $\begin{cases} 9x + 3y \le 6 \\ y > x \end{cases}$

Graph each system of linear inequalities.

23. $\begin{cases} y < 3 \\ y > 5 \end{cases}$

24. $\begin{cases} y < x - 1 \\ y > x - 2 \end{cases}$

25. $\begin{cases} x \ge 2 \\ x \le 2 \end{cases}$

26. $\begin{cases} y > -4x - 3 \\ y < -4x + 2 \end{cases}$

27. $\begin{cases} y > -1 \\ y > 2 \end{cases}$

28. $\begin{cases} y \le 2x + 1 \\ y \le 2x - 4 \end{cases}$

29. Multi-Step Linda works at a pharmacy for $15 an hour. She also baby-sits for $10 an hour. Linda needs to earn at least $90 per week, but she does not want to work more than 20 hours per week. Show and describe the number of hours Linda could work at each job to meet her goals. List two possible solutions.

30. Farming Tony wants to plant at least 40 acres of corn and at least 50 acres of soybeans. He wants no more than 200 acres of corn and soybeans. Show and describe all the possible combinations of the number of acres of corn and of soybeans Tony could plant. List two possible combinations.

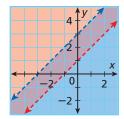**Graph each system of linear inequalities.**

31. $\begin{cases} y \geq -3 \\ y \geq 2 \end{cases}$ **32.** $\begin{cases} y > -2x - 1 \\ y > -2x - 3 \end{cases}$ **33.** $\begin{cases} x \leq -3 \\ x \geq 1 \end{cases}$ **34.** $\begin{cases} y < 4 \\ y > 0 \end{cases}$
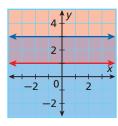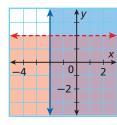
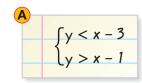**Write a system of linear inequalities to represent each graph.**

35.

36.
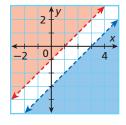
37.

38. Military For males to enter the United States Air Force Academy, located in Colorado Springs, CO, they must be at least 17 but less than 23 years of age. Their standing height must be not less than 60 inches and not greater than 80 inches. Graph all possible heights and ages for eligible male candidates. Give three possible combinations.

39. /// ERROR ANALYSIS /// Two students wrote a system of linear inequalities to describe the graph. Which student is incorrect? Explain the error.

A
$\begin{cases} y < x - 3 \\ y > x - 1 \end{cases}$

B
$\begin{cases} y > x - 3 \\ y < x - 1 \end{cases}$

40. Recreation Vance wants to fence in a rectangular area for his dog. He wants the length of the rectangle to be at least 30 feet and the perimeter to be no more than 150 feet. Graph all possible dimensions of the rectangle.

41. Critical Thinking Can the solutions of a system of linear inequalities be the points on a line? Explain.

42. This problem will prepare you for the Multi-Step Test Prep on page 428.

Gloria is starting her own company making teddy bears. She has enough bear bodies to create 40 bears. She will make girl bears and boy bears.

a. Write an inequality to show this situation.

b. Gloria will charge $15 for girl bears and $12 for boy bears. She wants to earn at least $540 a week. Write an inequality to describe this situation.

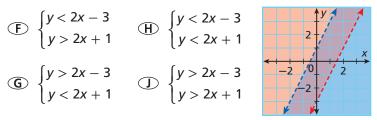**c.** Graph this situation and locate the solution region.

43. Write About It What must be true of the boundary lines in a system of two linear inequalities if there is no solution of the system? Explain.

44. Which point is a solution of $\begin{cases} 2x + y \geq 3 \\ y \geq -2x + 1 \end{cases}$?

A $(0, 0)$ $\qquad$ B $(0, 1)$ $\qquad$ C $(1, 0)$ $\qquad$ D $(1, 1)$

45. Which system of inequalities best describes the graph?

F $\begin{cases} y < 2x - 3 \\ y > 2x + 1 \end{cases}$ $\qquad$ H $\begin{cases} y < 2x - 3 \\ y < 2x + 1 \end{cases}$

G $\begin{cases} y > 2x - 3 \\ y < 2x + 1 \end{cases}$ $\qquad$ J $\begin{cases} y > 2x - 3 \\ y > 2x + 1 \end{cases}$

46. Short Response Graph and describe $\begin{cases} y + x > 2 \\ y \leq -3x + 4 \end{cases}$. Give two possible solutions of the system.

CHALLENGE AND EXTEND

47. Estimation Graph the given system of inequalities. Estimate the area of the overlapping solution regions.
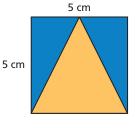
$\begin{cases} y \geq 0 \\ y \leq x + 3.5 \\ y \leq -x + 3.5 \end{cases}$

48. Write a system of linear inequalities for which $(-1, 1)$ and $(1, 4)$ are solutions and $(0, 0)$ and $(2, -1)$ are not solutions.

49. Graph $|y| < 1$.

50. Write a system of linear inequalities for which the solutions are all the points in the third quadrant.

SPIRAL REVIEW

Use the diagram to find each of the following. *(Lesson 1-6)*

51. area of the square

52. area of the yellow triangle

53. combined area of the blue triangles

Tell whether the given ordered pairs satisfy a linear function. *(Lesson 5-1)*

54. $\{(3, 8), (4, 6), (5, 4), (6, 2), (7, 0)\}$ $\qquad$ **55.** $\{(6, 1), (7, 2), (8, 4), (9, 7), (10, 11)\}$

56. $\{(2, 10), (7, 9), (12, 8), (17, 7), (22, 6)\}$ $\qquad$ **57.** $\{(1, -9), (3, -7), (5, -5), (7, -3), (9, -1)\}$

Graph the solutions of each linear inequality. *(Lesson 6-5)*

58. $y \leq 2x - 1$ $\qquad$ **59.** $-\dfrac{1}{4}x + y > 6$ $\qquad$ **60.** $5 - x \geq 0$

Solve Systems of Linear Inequalities

A graphing calculator gives a visual solution to a system of linear inequalities.

Use with Lesson 6-6

Activity

Graph the system $\begin{cases} y > 2x - 4 \\ 2.75y - x < 6 \end{cases}$. Give two ordered pairs that are solutions.
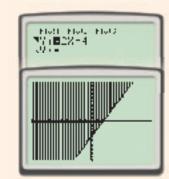
1 Write the first boundary line in slope-intercept form.

$y > 2x - 4 \qquad \longrightarrow \qquad y = 2x - 4$

2 Press [Y=] and enter $2x - 4$ for **Y1.**

The inequality contains the symbol $>$. The solution region is above the boundary line. Press [◄] to move the cursor to the left of **Y1.** Press [ENTER] until the icon that looks like a region above a line appears. Press [GRAPH].

3 Solve the second inequality for y.

$2.75y - x < 6$

$2.75y < x + 6$

$y < \dfrac{x + 6}{2.75} \qquad \longrightarrow \qquad y = \dfrac{x + 6}{2.75}$

4 Press [Y=] and enter $(x + 6)/2.75$ for **Y2.**

The inequality contains the symbol $<$. The solution region is below the boundary line. Press [◄] to move the cursor to the left of **Y2.** Press [ENTER] until the icon that looks like a region below a line appears. Press [GRAPH].

5 The solutions of the system are represented by the overlapping shaded regions. The points $(0, 0)$ and $(-1, 0)$ are in the shaded region.

Check Test $(0, 0)$ in both inequalities.   Test $(-1, 0)$ in both inequalities.

$y > 2x - 4$		$2.75y - x < 6$	
0	$2(0) - 4$	$2.75(0) - 0$	6
0	> -4 ✓	$0 < 6$ ✓	

$y > 2x - 4$		$2.75y - x < 6$	
0	$2(-1) - 4$	$2.75(0) - (-1)$	6
0	> -6 ✓	$1 < 6$ ✓	

Try This

Graph each system. Give two ordered pairs that are solutions.

1. $\begin{cases} x + 5y > -10 \\ x - y < 4 \end{cases}$ **2.** $\begin{cases} y > x - 2 \\ y \le x + 2 \end{cases}$ **3.** $\begin{cases} y > x - 2 \\ y \le 3 \end{cases}$ **4.** $\begin{cases} y < x - 3 \\ y - 3 > x \end{cases}$

MULTI-STEP TEST PREP

Equations and Formulas

Bearable Sales Gloria makes teddy bears. She dresses some as girl bears with dresses and bows and some as boy bears with bow ties. She is running low on supplies. She has only 100 eyes, 30 dresses, and 60 ties that can be used as bows on the girls and bow ties on the boys.

1. Write the inequalities that describe this situation. Let *x* represent the number of boy bears and *y* represent the number of girl bears.

2. Graph the inequalities and locate the region showing the number of boy and girl bears Gloria can make.

3. List at least three combinations of girl and boy bears that Gloria can make.

For 4 and 5, use the table.

4. Using the boundary line in your graph from Problem 2, copy and complete the table with the corresponding number of girl bears.

5. Gloria sells the bears for profit. She makes a profit of $8 for the girl bears and $5 for the boy bears. Use the table from Problem 4 to find the profit she makes for each given combination.

6. Which combination is the most profitable? Explain. Where does it lie on the graph?

Bear Combinations	
Boy	Girl
0	▪
10	▪
20	▪
30	▪
40	▪
50	▪

READY TO GO ON?

Quiz for Lessons 6-5 Through 6-6

✔ 6-5 Solving Linear Inequalities

Tell whether the ordered pair is a solution of the inequality.

1. $(3, -2); y < -2x + 1$ **2.** $(2, 1); y \geq 3x - 5$ **3.** $(1, -6); y \leq 4x - 10$

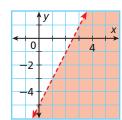**Graph the solutions of each linear inequality.**

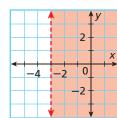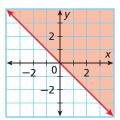**4.** $y \geq 4x - 3$ **5.** $3x - y < 5$ **6.** $2x + 3y < 9$ **7.** $y \leq -\frac{1}{2}x$

8. Theo's mother has given him at most \$150 to buy clothes for school. The pants cost \$30 each and the shirts cost \$15 each. How many of each can he buy? Write a linear inequality to describe the situation. Graph the linear inequality and give three possible combinations of pants and shirts Theo could buy.

Write an inequality to represent each graph.

9.

10.

11.

✔ 6-6 Solving Systems of Linear Inequalities

Tell whether the ordered pair is a solution of the given system.

12. $(-3, -1); \begin{cases} y > -2 \\ y < x + 4 \end{cases}$ **13.** $(-3, 0); \begin{cases} y \leq x + 4 \\ y \geq -2x - 6 \end{cases}$ **14.** $(0, 0); \begin{cases} y \geq 3x \\ 2x + y < -1 \end{cases}$

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

15. $\begin{cases} y > -2 \\ y < x + 3 \end{cases}$ **16.** $\begin{cases} x + y \leq 2 \\ 2x + y \geq -1 \end{cases}$ **17.** $\begin{cases} 2x - 5y \leq -5 \\ 3x + 2y < 10 \end{cases}$

Graph each system of linear inequalities and describe the solutions.

18. $\begin{cases} y \geq x + 1 \\ y \geq x - 4 \end{cases}$ **19.** $\begin{cases} y \geq 2x - 1 \\ y < 2x - 3 \end{cases}$ **20.** $\begin{cases} y < -3x + 5 \\ y > -3x - 2 \end{cases}$

21. A grocer sells mangos for \$4/lb and apples for \$3/lb. The grocer starts with 45 lb of mangos and 50 lb of apples each day. The grocer's goal is to make at least \$300 by selling mangos and apples each day. Show and describe all possible combinations of mangos and apples that could be sold to meet the goal. List two possible combinations.

Study Guide: Review

Vocabulary

Complete the sentences below with vocabulary words from the list above.

1. A(n) ____?____ is a system that has exactly one solution.

2. A set of two or more linear equations that contain the same variable(s) is
a(n) ____?____ .

3. The ____?____ consists of all the ordered pairs that satisfy all the inequalities
in the system.

4. A system consisting of equations of parallel lines with different y-intercepts
is a(n) ____?____ .

5. A(n) ____?____ consists of two intersecting lines.
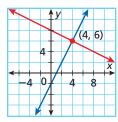
6-1 Solving Systems by Graphing (pp. 383–388)

EXAMPLE

■ Solve $\begin{cases} y = 2x - 2 \\ x + 2y = 16 \end{cases}$ by graphing.

Check your answer.

$\begin{cases} y = 2x - 2 \\ y = -\dfrac{1}{2}x + 8 \end{cases}$ *Write the second equation in slope-intercept form.*

The solution appears to be at $(4, 6)$.

$y = 2x - 2$		$x + 2y = 16$	
6	$2(4) - 2$	$4 + 2(6)$	16
6	6 ✓	16	16 ✓

The ordered pair $(4, 6)$ makes both equations
true, so it is a solution of the system.

EXERCISES

Tell whether the ordered pair is a solution of the given system.

6. $(0, -5); \begin{cases} y = -6x + 5 \\ x - y = 5 \end{cases}$

7. $(4, 3); \begin{cases} x - 2y = -2 \\ y = \dfrac{1}{2}x + 1 \end{cases}$

8. $\left(1\dfrac{3}{4}, 7\dfrac{1}{4}\right); \begin{cases} x + y = 9 \\ 2y = 6x + 4 \end{cases}$

9. $(-1, -1); \begin{cases} y = -2x + 5 \\ 3y = 6x + 3 \end{cases}$

Solve each system by graphing. Check your answer.

10. $\begin{cases} y = 3x + 2 \\ y = -2x - 3 \end{cases}$

11. $\begin{cases} y = -\dfrac{1}{3}x + 5 \\ 2x - 2y = -2 \end{cases}$

12. Raheel is comparing the cost of two parking
garages. Garage A charges a flat fee of $6 per car
plus $0.50 per hour. Garage B charges a flat fee of
$2 per car plus $1 per hour. After how many hours
will the cost at garage A be the same as the cost at
garage B? What will that cost be?

6-2 Solving Systems by Substitution (pp. 390–396)

EXAMPLE

■ Solve $\begin{cases} 2x - 3y = -2 \\ y - 3x = 10 \end{cases}$ by substitution.

Step 1 $y - 3x = 10$ *Solve the second*
$y = 3x + 10$ *equation for y.*

Step 2 $2x - 3y = -2$ *Substitute $3x + 10$*
$2x - 3(3x + 10) = -2$ *for y in the first equation.*

Step 3 $2x - 9x - 30 = -2$ *Solve for x.*
$-7x - 30 = -2$
$-7x = 28$
$x = -4$

Step 4 $y - 3x = 10$ *Substitute -4 for x.*
$y - 3(-4) = 10$
$y + 12 = 10$ *Find the value of y.*
$y = -2$

Step 5 $(-4, -2)$ *Write the solution as an ordered pair.*

To check the solution, substitute $(-4, -2)$ into both equations in the system.

EXERCISES

Solve each system by substitution.

13. $\begin{cases} y = x + 3 \\ y = 2x + 12 \end{cases}$ **14.** $\begin{cases} y = -4x \\ y = 2x - 3 \end{cases}$

15. $\begin{cases} 2x + y = 4 \\ 3x + y = 3 \end{cases}$ **16.** $\begin{cases} x + y = -1 \\ y = -2x + 3 \end{cases}$

17. $\begin{cases} x = y - 7 \\ -y - 2x = 8 \end{cases}$ **18.** $\begin{cases} \frac{1}{2}x + y = 9 \\ 3x - 4y = -6 \end{cases}$

19. The Nash family's car needs repairs. Estimates for parts and labor from two garages are shown below.

Garage	Parts ($)	Labor ($ per hour)
Motor Works	650	70
Jim's Car Care	800	55

For how many hours of labor will the total cost of fixing the car be the same at both garages? What will that cost be? Which garage will be cheaper if the repairs require 8 hours of labor? Explain.

6-3 Solving Systems by Elimination (pp. 397–403)

EXAMPLE

■ Solve $\begin{cases} 2x - 3y = -8 \\ x + 4y = 7 \end{cases}$ by elimination.

Step 1   *Multiply the second equation by -2.*
$2x - 3y = -8$
$+(-2)(x + 4y = 7)$
$2x - 3y = -8$ *Eliminate the x-term.*
$+(-2x - 8y = -14)$

Step 2 $0x - 11y = -22$ *Solve for y.*
$y = 2$

Step 3 $2x - 3y = -8$
$2x - 3(2) = -8$ *Substitute 2 for y.*
$2x - 6 = -8$ *Simplify and solve*
$2x = -2$ *for x.*
$x = -1$

Step 4 $(-1, 2)$ *Write the solution as an ordered pair.*

To check the solution, substitute $(-1, 2)$ into both equations in the system.

EXERCISES

Solve each system by elimination.

20. $\begin{cases} 4x + y = -1 \\ 2x - y = -5 \end{cases}$ **21.** $\begin{cases} x + 2y = -1 \\ x + y = 2 \end{cases}$

22. $\begin{cases} x + y = 12 \\ 2x + 5y = 27 \end{cases}$ **23.** $\begin{cases} 3x - 2y = -6 \\ \frac{1}{3}x + 3y = 9 \end{cases}$

Solve each system by any method. Explain why you chose each method. Check your answer.

24. $\begin{cases} 3x + y = 2 \\ y = -4x \end{cases}$ **25.** $\begin{cases} y = \frac{1}{3}x - 6 \\ y = -2x + 1 \end{cases}$

26. $\begin{cases} 2y = -3x \\ y = -2x + 2 \end{cases}$ **27.** $\begin{cases} x - y = 0 \\ 3x + y = 8 \end{cases}$

EXAMPLES

Classify each system. Give the number of solutions.

■ $\begin{cases} y = 3x + 4 \\ 6x - 2y = -8 \end{cases}$

Use the substitution method because the first equation is solved for y.

$6x - 2(3x + 4) = -8$ *Substitute $3x + 4$ for y in*
$6x - 6x - 8 = -8$ *the second equation.*
$-8 = -8$ ✓   *True.*

The equation is an identity. There are infinitely many solutions.

This system is **consistent** and **dependent**. The two lines are coincident (the same line) because they have identical slopes and y-intercepts.

■ $\begin{cases} y = 2x - 1 \\ 2x - y = -2 \end{cases}$

Compare slopes and y-intercepts. Write both equations in slope-intercept form.

$\begin{cases} y = 2x - 1 & \Rightarrow y = 2x - 1 \\ 2x - y = -2 & \Rightarrow y = 2x + 2 \end{cases}$

The lines have the same slope and different y-intercepts. The lines are parallel.

The lines never intersect, so this system is **inconsistent**. It has **no solution.**

■ $\begin{cases} 2x - y = 6 \\ y = x - 1 \end{cases}$

Write both equations in slope-intercept form.

$\begin{cases} 2x - y = 6 & \Rightarrow y = 2x - 6 \\ y = x - 1 & \Rightarrow y = 1x - 1 \end{cases}$

The lines intersect because they have different slopes.

The system is **consistent** and **independent.** There is **one solution:** $(5, 4)$.

EXERCISES

Solve each system of linear equations.

28. $\begin{cases} y = \frac{1}{4}x - 3 \\ y = \frac{1}{4}x + 5 \end{cases}$ **29.** $\begin{cases} y = -x + 4 \\ x + y = 4 \end{cases}$

30. $\begin{cases} y = 3x + 2 \\ y = 2x \end{cases}$ **31.** $\begin{cases} -4x - y = 6 \\ \frac{1}{2}y = -2x - 3 \end{cases}$

32. $\begin{cases} x + 2y = 8 \\ y = -\frac{1}{2}x + 4 \end{cases}$ **33.** $\begin{cases} y - 2x = -1 \\ y + 2x = -5 \end{cases}$

34. Tristan and his friend Marco just started DVD collections. They continue to get DVDs at the rate shown in the table below. Will Tristan ever have the same number of DVDs as Marco? Explain.

DVD Collections			
	Month 1	**Month 2**	**Month 3**
Tristan	2	7	12
Marco	8	12	16

Classify each system. Give the number of solutions.

35. $\begin{cases} y = \frac{1}{2}x + 2 \\ y = \frac{1}{4}x - 8 \end{cases}$ **36.** $\begin{cases} y = 3x - 7 \\ y = 3x + 2 \end{cases}$
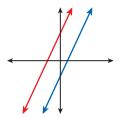
37. $\begin{cases} 2x + y = 2 \\ y - 2 = -2x \end{cases}$ **38.** $\begin{cases} -3x - y = -5 \\ y = -3x - 5 \end{cases}$

39. $\begin{cases} 2x + 3y = 1 \\ 3x + 2y = 1 \end{cases}$ **40.** $\begin{cases} x + \frac{1}{2}y = 3 \\ 2x = 6 - y \end{cases}$

41. The two parallel lines graphed below represent a system of equations. Classify the system and give the number of solutions.

6-5 Solving Linear Inequalities (pp. 414–420)

EXAMPLE

■ Graph the solutions of $x - 2y < 6$.

Step 1 Solve the inequality for y.
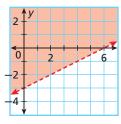$$x - 2y < 6$$
$$-2y < -x + 6$$
$$y > \frac{1}{2}x - 3$$

Step 2 Graph $y = \frac{1}{2}x - 3$.
Use a dashed line for $>$.

Step 3 The inequality is $>$, so shade above the boundary line.

Check Substitute $(0, 0)$ for (x, y) because it is not on the boundary line.

$$\begin{array}{r|l} x - 2y < 6 \\ \hline 0 - 2(0) \;\big|\; 6 \\ 0 < 6 ✓ \end{array}$$

(0, 0) satisfies the inequality, so the graph is shaded correctly.

EXERCISES

Tell whether the ordered pair is a solution of the inequality.

42. $(0, -3); y < 2x - 3$

43. $(2, -1); y \geq x - 3$

44. $(6, 0); y > -3x + 4$

45. $(10, 10); y \leq x - 3$

Graph the solutions of each linear inequality.

46. $y < -2x + 5$ **47.** $x - y \geq 2$

48. $-x + 2y \geq 6$ **49.** $y > -4x$

50. $x + y + 4 > 0$ **51.** $5 - y \geq 2x$

52. The Mathematics Club is selling pizza and lemonade to raise money for a trip. They estimate that the trip will cost at least $450. If they make $2 on each slice of pizza and $1 on each bottle of lemonade, how many of each do they need to sell to have enough money for their trip? Write an inequality to describe the situation. Graph and then give two combinations of the number of pizza slices and number of lemonade bottles they need to sell.

6-6 Solving Systems of Linear Inequalities (pp. 421–426)

EXAMPLES

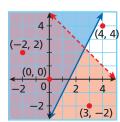■ Graph $\begin{cases} y < -x + 5 \\ y \geq 2x - 3 \end{cases}$. Give two ordered pairs that are solutions and two that are not solutions.

Graph both inequalities.

The solutions of the system are represented by the overlapping shaded regions.

The points $(0, 0)$ and $(-2, 2)$ are solutions of the system.

The points $(3, -2)$ and $(4, 4)$ are not solutions.

EXERCISES

Tell whether the ordered pair is a solution of the given system.

53. $(3, 3); \begin{cases} y > -2x + 9 \\ y \geq x \end{cases}$ **54.** $(-1, 0); \begin{cases} 2x - y > -5 \\ y \leq -3x - 3 \end{cases}$

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

55. $\begin{cases} y \geq x + 4 \\ y > 6x - 3 \end{cases}$ **56.** $\begin{cases} y \leq -2x + 8 \\ y > 3x - 5 \end{cases}$

57. $\begin{cases} -x + 2y > 6 \\ x + y < 4 \end{cases}$ **58.** $\begin{cases} x - y > 7 \\ x + 3y \leq 15 \end{cases}$

Graph each system of linear inequalities.

59. $\begin{cases} y > -x - 6 \\ y < -x + 5 \end{cases}$ **60.** $\begin{cases} 4x + 2y \geq 10 \\ 6x + 3y < -9 \end{cases}$

CHAPTER TEST

Tell whether the ordered pair is a solution of the given system.

1. $(1, -4);$ $\begin{cases} y = -4x \\ y = 2x - 2 \end{cases}$

2. $(0, -1);$ $\begin{cases} 3x - y = 1 \\ x + 5y = -5 \end{cases}$

3. $(3, 2);$ $\begin{cases} x - 2y = -1 \\ -3x + 2y = 5 \end{cases}$

Solve each system by graphing.

4. $\begin{cases} y = x - 3 \\ y = -2x - 3 \end{cases}$

5. $\begin{cases} 2x + y = -8 \\ y = \frac{1}{3}x - 1 \end{cases}$

6. $\begin{cases} y = -x + 4 \\ x = y + 2 \end{cases}$

Solve each system by substitution.

7. $\begin{cases} y = -6 \\ y = -2x - 2 \end{cases}$

8. $\begin{cases} -x + y = -4 \\ y = 2x - 11 \end{cases}$

9. $\begin{cases} x - 3y = 3 \\ 2x = 3y \end{cases}$

10. The costs for services at two kennels are shown in the table. Joslyn plans to board her dog and have him bathed once during his stay. For what number of days will the cost for boarding and bathing her dog at each kennel be the same? What will that cost be? If Joslyn plans a week-long vacation, which is the cheaper service? Explain.

Kennel Costs		
	Boarding ($ per day)	Bathing ($)
Pet Care	30	15
Fido's	28	27

Solve each system by elimination.

11. $\begin{cases} 3x - y = 7 \\ 2x + y = 3 \end{cases}$

12. $\begin{cases} 4x + y = 0 \\ x + y = -3 \end{cases}$

13. $\begin{cases} 2x + y = 3 \\ x - 2y = -1 \end{cases}$

Classify each system. Give the number of solutions.

14. $\begin{cases} y = 6x - 1 \\ 6x - y = 1 \end{cases}$

15. $\begin{cases} y = -3x - 3 \\ 3x + y = 3 \end{cases}$

16. $\begin{cases} 2x - y = 1 \\ -4x + y = 1 \end{cases}$

Graph the solutions of each linear inequality.

17. $y < 2x - 5$

18. $-y \geq 8$

19. $y > \frac{1}{3}x$

Graph each system of linear inequalities. Give two ordered pairs that are solutions and two that are not solutions.

20. $\begin{cases} y > \frac{1}{2}x - 5 \\ y \leq 4x - 1 \end{cases}$

21. $\begin{cases} y > -x + 4 \\ 3x - y > 3 \end{cases}$

22. $\begin{cases} y \geq 2x \\ y - 2x < 6 \end{cases}$

23. Ezra and Tava sold at least 150 coupon books. Ezra sold at most 30 books more than twice the number Tava sold. Show and describe all possible combinations of the numbers of coupon books Ezra and Tava sold. List two possible combinations.
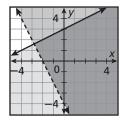
COLLEGE ENTRANCE EXAM PRACTICE

FOCUS ON ACT

Four scores are reported for the ACT Mathematics Test: one score based on all 60 problems and one for each content area. The three content areas are: Pre-Algebra/Elementary Algebra, Intermediate Algebra/Coordinate Geometry, and Plane Geometry/Trigonometry.

HOT TIP! Taking classes that cover the content areas on the ACT is a good idea. This way you will have skills from each area of the test. Preparation over a long term is better than cramming at the last minute.

You may want to time yourself as you take this practice test. It should take you about 5 minutes to complete.

1. Which system of inequalities is represented by the graph?

(A) $\begin{cases} -x + 2y < 6 \\ 2x + y > -4 \end{cases}$

(B) $\begin{cases} x - 2y \le 6 \\ 2x - y \ge 4 \end{cases}$

(C) $\begin{cases} -x + 2y \le 6 \\ 2x + y \ge 4 \end{cases}$

(D) $\begin{cases} -x + 2y \le 6 \\ 2x + y > -4 \end{cases}$

(E) $\begin{cases} x - 2y \le 6 \\ 2x - y > 4 \end{cases}$

2. What is the solution for y in the given system?

$\begin{cases} 4x + 3y = 1 \\ -4x + 3y = -7 \end{cases}$

(F) -1

(G) 0

(H) 1

(J) 2

(K) 6

3. Wireless phone company A charges $20 per month plus $0.12 per minute. Wireless phone company B charges $50 per month plus $0.06 per minute. For how many minutes of calls will the monthly bills be the same?

(A) 80 minutes

(B) 100 minutes

(C) 160 minutes

(D) 250 minutes

(E) 500 minutes

4. Which of the following systems of equations does NOT have a solution?

(F) $\begin{cases} x + 5y = 30 \\ -4x + 5y = 10 \end{cases}$

(G) $\begin{cases} x + 5y = -30 \\ -4x + 5y = 10 \end{cases}$

(H) $\begin{cases} x + 5y = -30 \\ -4x + 5y = -10 \end{cases}$

(J) $\begin{cases} -4x + 5y = -10 \\ -8x + 10y = -20 \end{cases}$

(K) $\begin{cases} -4x + 5y = -10 \\ -4x + 5y = -30 \end{cases}$

TEST TACKLER

Standardized Test Strategies

Any Question Type: Read the Problem for Understanding

Standardized test questions may vary in format including multiple choice, gridded response, and short or extended response. No matter what format the test uses, read each question carefully and critically. Do not rush. Be sure you completely understand what you are asked to do and what your response should include.

EXAMPLE **1**

Extended Response

An interior decorator charges a consultation fee of $50 plus $12 per hour. Another interior decorator charges a consultation fee of $5 plus $22 per hour. Write a system of equations to find the amount of time for which the cost of both decorators will be the same. Graph the system. After how many hours will the cost be the same for both decorators? What will the cost be?

Read the problem again.

What information are you given?

the consultation fees and hourly rates of two decorators

What are you asked to do?

1. Write a system of equations.
2. Graph the system.
3. Interpret the solution to the system.

What should your response include?

1. a system of equations with variables defined
2. a graph of the system
3. the time when the cost is the same for both decorators
4. the cost at that time

Read each test item and answer the questions that follow.

Item A
Short Response Which value of b will make the lines intersect at the point $(-2, 14)$?

$$\begin{cases} y = -6x + 2 \\ y = 4x + b \end{cases}$$

1. What information are you given?

2. What are you asked to do?

3. Ming's answer to this test problem was $y = 4x + 22$. Did Ming answer correctly? Explain.

Item B
Extended Response Solve the system by using elimination. Explain how you can check your solution algebraically and graphically.

$$\begin{cases} 4x + 10y = -48 \\ 6x - 10y = 28 \end{cases}$$

4. What method does the problem ask you to use to solve the system of equations?

5. What methods does the problem ask you to use to check your solution?

6. How many parts are there to this problem? List what needs to be included in your response.

Item C
Gridded Response What is the x-coordinate of the solution to this

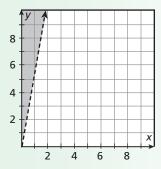system? $\begin{cases} y = 6x + 9 \\ y = 12x - 15 \end{cases}$

7. What question is being asked?

8. A student correctly found the solution of the system to be $(4, 33)$. What should the student mark on the grid so that the answer is correct?

Item D
Short Response Write an inequality to represent the graph below. Give a real-world situation that this inequality could describe.

9. As part of his answer, a student wrote the following response:

> The point $(1, 5)$ is not a solution to the inequality because it lies on the line, but $(2, 12)$ is a solution because it lies above the line.

Is his response appropriate? Explain.

10. What should the response include so that it answers all parts of the problem?

Item E
Multiple Choice Taylor bikes 50 miles per week and increases her distance by 2 miles each week. Josie bikes 30 miles per week and increases her distance by 10 miles each week. In how many weeks will Taylor and Josie be biking the same distance?

Ⓐ 2.5 weeks   Ⓒ 55 weeks

Ⓑ 7.5 weeks   Ⓓ 110 weeks

11. What question is being asked?

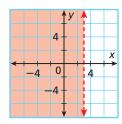12. Carson incorrectly selected option C as his answer. What question did he most likely answer?

CUMULATIVE ASSESSMENT, CHAPTERS 1–6

Multiple Choice

1. If $a > 0$ and $b < 0$, which of the following must be true?

Ⓐ $ab > 0$ Ⓒ $a + b > 0$

Ⓑ $ab < 0$ Ⓓ $a + b < 0$

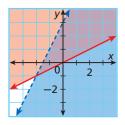**2.** Which of the problems below could be solved by finding the solution of this system?

$$\begin{cases} 2x + 2y = 56 \\ y = \dfrac{1}{3}x \end{cases}$$

Ⓕ The area of a rectangle is 56. The width is one-third the length. Find the length of the rectangle.

Ⓖ The area of a rectangle is 56. The length is one-third the perimeter. Find the length of the rectangle.

Ⓗ The perimeter of a rectangle is 56. The length is one-third more than the width. Find the length of the rectangle.

Ⓙ The perimeter of a rectangle is 56. The width is one-third the length. Find the length of the rectangle.

3. What is the slope of a line perpendicular to a line that passes through $(3, 8)$ and $(1, -4)$?

Ⓐ $-\dfrac{1}{6}$ Ⓒ 2

Ⓑ $-\dfrac{1}{2}$ Ⓓ 6

4. Which inequality is graphed below?

Ⓕ $-x > -3$ Ⓗ $2x < -6$

Ⓖ $-y > -3$ Ⓙ $3y < 9$

5. A chemist has a bottle of a 10% acid solution and a bottle of a 30% acid solution. He mixes the solutions together to get 500 mL of a 25% acid solution. How much of the 30% solution did he use?

Ⓐ 125 mL   Ⓒ 375 mL

Ⓑ 150 mL   Ⓓ 450 mL

6. Which ordered pair is NOT a solution of the system graphed below?

Ⓕ $(0, 0)$ Ⓗ $(1, 1)$

Ⓖ $(0, 3)$ Ⓙ $(2, 1)$

7. Which of the following best classifies a system of linear equations whose graph is two intersecting lines?

Ⓐ inconsistent and dependent

Ⓑ inconsistent and independent

Ⓒ consistent and dependent

Ⓓ consistent and independent

8. Which ordered pair is a solution of this system?

$$\begin{cases} 2x - y = -2 \\ \dfrac{1}{3}y = x \end{cases}$$

Ⓕ $(0, 2)$ Ⓗ $(2, 6)$

Ⓖ $(1, 3)$ Ⓙ $(3, 8)$

9. Where does the graph of $5x - 10y = 30$ cross the $y-$axis?

Ⓐ $(0, -3)$ Ⓒ $(6, 0)$

Ⓑ $\left(0, \dfrac{1}{2}\right)$ Ⓓ $(0, -6)$

10. Hillary needs markers and poster board for a project. The markers are $0.79 each and the poster board is $1.89 per sheet. She needs at least 4 sheets of poster board. Hillary has $15 to spend on project materials. Which system models this information?

(F) $\begin{cases} p \geq 4 \\ 0.79m + 1.89p \leq 15 \end{cases}$

(G) $\begin{cases} 0.79m \geq 1.89p \\ 4p \leq 15 \end{cases}$
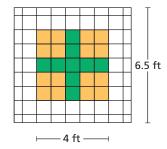
(H) $\begin{cases} 4p \geq 1.89 \\ m + 4p \leq 15 \end{cases}$

(J) $\begin{cases} p + m \leq 15 \\ 0.79m + 1.89p \geq 4 \end{cases}$

11. Find the slope of the line described by $4x - 2y = -8$.

(A) -2 (C) 2

(B) $-\frac{1}{2}$ (D) 4

Gridded Response

12. The floor in the entranceway of Kendra's house is square and measures 6.5 feet on each side. Colored tiles have been set in the center of the floor in a square measuring 4 feet on each side. The remaining floor consists of white tiles.

6.5 ft

4 ft

How many square feet of the entranceway floor consists of white tiles?

13. What value of y will make the line passing through $(4, -4)$ and $(-8, y)$ have a slope of $-\frac{1}{2}$?

14. What value of k will make the system $y - 5x = -1$ and $y = kx + 3$ inconsistent?

Short Response

15. The data in the table represents a linear relationship between x and y. Find the missing y-value. Explain your answer.

x	y
−3	−15
−1	−7
0	−3
2	▣
5	17

16. Graph $y > \frac{-x}{3} - 1$ on a coordinate plane. Name one point that is a solution of the inequality.

17. Marc and his brother Ty start saving money at the same time. Marc has $145 and will add $10 to his savings every week. Ty has $20 and will add $15 to his savings every week. After how many weeks will Marc and Ty have the same amount saved? What is that amount? Show your work.

18. A movie producer is looking for extras to act as office employees in his next movie. The producer needs extras that are at least 40 years old but less than 70 years old. They should be at least 60 inches tall but less than 75 inches tall. Graph all the possible combinations of ages and heights for extras that match the producer's needs. Let x represent age and y represent height. Show your work.

19. Graph the system $\begin{cases} y < -2x + 3 \\ y \geq 6x + 6 \end{cases}$.

a. Is $(0, 0)$ a solution of the system you graphed? Explain why or why not.

b. Is $(-4, 5)$ a solution of the system you graphed? Explain why or why not.

Extended Response

20. Every year, Erin knits scarves and sells them at the craft fair. This year she used $6 worth of yarn for each scarf. She also paid $50 to rent a table at the fair. She sold every scarf for $10.

a. Write a system of linear equations to represent the amount Erin spent and the amount she collected. Tell what your variables represent. Tell what each equation in the system represents.

b. Use any method to solve the system you wrote in part **a.** Show your work. How many scarves did Erin need to sell to make a profit? Explain.

c. Describe two ways you could check your solution to part **b.** Check your solution by using one of those ways. Show your work.

Problem Solving on Location

NEW JERSEY

Pine Barrens

⭐ Kayaking and Canoeing in the Pine Barrens

The Pine Barrens, or Pinelands, of New Jersey cover over 1 million acres in southern and central New Jersey. Kayaking on the Wading and Oswego Rivers is a popular summertime activity. There are several campgrounds and canoe and kayak rentals located along the rivers.

Choose one or more strategies to solve each problem.

1. A group of kayakers starts at Evans Bridge and kayaks to Beaver Branch. Two other groups of kayakers start at Oswego Lake. One of these groups heads toward Harrisville, and the other heads toward Bodine Field. A fourth group starts at Speedwell and kayaks to Chips Folly. Draw a mapping diagram to represent the starting and ending points of the trips. Tell whether the relation is a function. Explain.

For 2 and 3, use the table.

2. If the average speed of canoes on the river is $1\frac{1}{2}$ miles per hour, about how far is it from Oswego Lake to Beaver Branch?

3. All river trips must be completed by 4:30 P.M. What is the latest time a canoetrip can start in order to make it from Speedwell to Evans Bridge?

Approximate Canoe Trip Times on the Wading and Oswego Rivers	
Trip	**Time (h)**
Evans Bridge to Beaver Branch	2
Hawkins Bridge to Evans Bridge	3
Oswego Lake to Harrisville	4
Oswego Lake to Beaver Branch	$4\frac{1}{2}$
Hawkins Bridge to Beaver Branch	5
Speedwell to Evans Bridge	6
Speedwell to Beaver Branch	8

Problem Solving Strategies

Draw a Diagram
Make a Model
Guess and Test
Work Backward
Find a Pattern
Make a Table
Solve a Simpler Problem
Use Logical Reasoning
Use a Venn Diagram
Make an Organized List

⭐ American Black Bear

American black bears are the most common bears in the United States. They can be found in 11 of the 21 counties in New Jersey. Most wild male black bears weigh between 125 and 600 pounds, while females generally weigh between 90 and 300 pounds. Their weight depends upon their age, the season of the year, and how much food is available.

Choose one or more strategies to solve each problem.

1. Black bears hibernate for about 5 months in the winter. They must store 50 to 60 pounds of fat to survive hibernation. During the month before hibernation, a black bear may consume up to 20,000 Calories per day. If a bear consumes an average of 18,500 Calories per day in the month of August, about how many total Calories will he consume that month?

For 2, use the graph.

2. A male bear weighed $\frac{2}{3}$ pound at birth. When he died at 20 years of age, he weighed 400 pounds. Use the graph to estimate how much the bear weighed when he was 7 years old.

3. A white-tailed deer is running from a black bear at 20 miles per hour. It is $\frac{1}{3}$ mile in front of the bear. The bear is running at 30 miles per hour. How long will it take the bear to catch the deer? Assume both continue running at that same pace.

4. During the hibernation months, the bear's heart rate slows to about 10 beats per minute. If the bear hibernates for 155 days, how many times will his heart beat?

Male Black Bear Weight

Weight (lb)

(20, 400)

$(0, \frac{2}{3})$

Age (y)